King Fahd University of Petroleum & Minerals

First Major Examination

Faculty: Science	**Department: Mathematics**
Semester: 181	**Course Name: Financial Mathematics**
Instructor: Abedalhay Elmughrabi	**Course No: AS 201**
Exam Date: October 31st, 2018	**Exam Time: 06:00 PM – 09:00 PM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks
1	4 points		11	4 points		21	4 points	
2	4 points		12	4 points		22	3 points	
3	4 points		13	4 points		23	3 points	
4	4 points		14	4 points		24	3 points	
5	4 points		15	4 points		25	3 points	
6	4 points		16	4 points				
7	4 points		17	4 points				
8	4 points		18	4 points				
9	4 points		19	4 points				
10	4 points		20	4 points				
Obtained Total: / 100								

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **25** questions.
3. Answers without calculations/steps will receive zero marks.
4. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
5. Calculators cannot be exchanged during the examination.
6. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
7. Cell phones must be turned off and placed under your bench facedown.
8. No questions are allowed during the exam.
9. All material related to the course should be put away
10. Be clean, neat and tidy, else your work may not be marked
11. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

Question 1 (4 Points):

Bruce deposits 100 into a bank account. His account is credited interest at an annual nominal rate of interest of 4% convertible semiannually.
At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at an annual force of interest of δ.
After 7.25 years, the value of each account is the same.
Calculate δ.

(A) 0.0388
(B) 0.0392
(C) 0.0396
(D) 0.0404
(E) 0.0414

Question 2 (4 Points):

Eric deposits 100 into a savings account at time 0, which pays interest at an annual nominal rate of i, compounded semiannually.
Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i.
Eric and Mike earn the same amount of interest during the last 6 months of the 8th year.
Calculate i.

(A) 9.06%
(B) 9.26%
(C) 9.46%
(D) 9.66%
(E) 9.86%

Question 3 (4 Points):

1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.
The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.
Calculate the accumulated value of Fund Y at the end of year 10.

(A) 1519
(B) 1819
(C) 2085
(D) 2273
(E) 2431

Question 4 (4 Points):

A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.
The annual effective rate of interest is 8%.
Calculate X.

(A) 54
(B) 64
(C) 74
(D) 84
(E) 94

Question 5 (4 Points):

Ernie makes deposits of 100 at time 0, and *X* at time 3. The fund grows at a force of interest, $\delta_t = \frac{t^2}{100}$, t > 0.
The amount of interest earned from time 3 to time 6 is also *X*.
Calculate *X*.

(A) 385
(B) 485
(C) 585
(D) 685
(E) 785

Question 6 (4 Points):

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.
Calculate the outstanding loan balance immediately after the 40^{th} payment is made.

(A) 6750
(B) 6890
(C) 6940
(D) 7030
(E) 7340

Question 7 (4 Points):

To accumulate 8000 at the end of 3*n* years, deposits of 98 are made at the end of each of the first *n* years and 196 at the end of each of the next 2*n* years.
The annual effective rate of interest is *i*. You are given $(1+i)^n = 2$
Calculate *i*.

(A) 11.25%
(B) 11.75%
(C) 12.25%
(D) 12.75%
(E) 13.25%

Question 8 (4 Points):

Olga buys a 5-year increasing annuity for X.
Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.
The annual nominal interest rate is 9% convertible quarterly.
Calculate X.

(A) 2680
(B) 2730
(C) 2780
(D) 2830
(E) 2880

Question 9 (4 Points):

A perpetuity-immediate pays *X* per year. Brian receives the first *n* payments, Colleen receives the next *n* payments, and a charity receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and the charity's share is *K*.
Calculate *K*.

(A) 24%
(B) 28%
(C) 32%
(D) 36%
(E) 40%

Question 10 (4 Points):

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.
At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.
Calculate X.

(A) 31.6
(B) 32.6
(C) 33.6
(D) 34.6
(E) 35.6

Question 11 (4 Points):

A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.
The annual effective interest rate is 5%.
Calculate X.

(A) 842
(B) 977
(C) 1017
(D) 1029
(E) 1075

Question 12 (4 Points):

Jack inherited a perpetuity-due, with annual payments of 15,000. He immediately exchanged the perpetuity for a 25-year annuity-due having the same present value. The annuity-due has annual payments of *X*.
All the present values are based on an annual effective interest rate of 10% for the first 10 years and 8% thereafter.
Calculate *X*.

(A) 16,942
(B) 17,384
(C) 17,434
(D) 17,520
(E) 18,989

Question 13 (4 Points):

A loan of 20,000 is repaid by a payment of X at the end of each year for 10 years. The loan has an annual effective interest rate of 11% for the first five years and 12% thereafter.
Calculate X.

(A) 2739.5
(B) 3078.5
(C) 3427.5
(D) 3467.5
(E) 3484.5

Question 14 (4 Points):

Ron has a loan with a present value of $a_{\overline{n}|i}$. The sum of the interest paid in period t plus the principal repaid in period $t + 1$ is X.
Calculate X.

(a) $1 + \frac{v^{n-t}}{i}$
(b) $1 + \frac{v^{n-t}}{d}$
(c) $1 + v^{n-t}i$
(d) $1 + v^{n-t}d$
(e) $1 + v^{n-t}$

Question 15 (4 Points):

At time t = 0, 1 is deposited into each of Fund X and Fund Y. Fund X accumulates at a force of interest $\delta_t = \frac{t^2}{k}$.
Fund Y accumulates at a nominal rate of discount of 8% per annum convertible semiannually.

At time t = 5, the accumulated value of Fund X equals the accumulated value of Fund Y .
Determine k.

(A) 100
(B) 102
(C) 104
(D) 106
(E) 108

Question 16 (4 Points):

At an annual effective interest rate of i, $i > 0$, both of the following annuities have a present value of X:
(i) a 20-year annuity-immediate with annual payments of 55
(ii) a 30-year annuity-immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years
Calculate X.

(A) 575
(B) 585
(C) 595
(D) 605
(E) 615

Question 17 (4 Points):

Jeff bought an increasing perpetuity due with annual payments starting at 5 and increasing by 5 each year until the payment reaches 100. The payments remain at 100 thereafter. The annual effective interest rate is 4.5%. Determine the present value of this perpetuity?

a. 1530
b. 1555
c. 1580
d. 1610
e. 1565

Question 18 (4 Points):

Fund A is invested at an effective annual interest rate of 3%.
Fund B is invested at an effective interest rate of 2.5%.
At the end of 20 years, the total in the two funds is 10,000. At the end of 31 years, the amount in fund A is twice the amount in fund B.
Calculate the total in the two funds at the end of 10 years.

a. 5,732
b. 6,602
c. 7,472
d. 7,569
e. 8,123

Question 19 (4 Points):

Jennifer deposits 1000 into a bank account. The bank credits interest at a nominal annual rate of i convertible semiannually for the first 7 years and a nominal annual rate of 2i convertible quarterly for all years thereafter.

The accumulated amount in the account at the end of 5 years is X. The accumulated amount in the account at the end of 10.5 years is 1980. Calculate X.

a. 1200
b. 1225
c. 1250
d. 1275
e. 1300

Question 20 (4 Points):

Mary deposits $15,000 today in a bank crediting interest at a nominal rate of 5% compounded monthly. This deposit is just sufficient to permit her to make monthly withdrawals of X for 6 years, first withdrawal today. Determine X.

a. 240.7
b. 152.3
c. 270.1
d. 310.5
e. 425

Question 21 (4 Points):

Andrea makes deposits of $100 on the first day of each month in calendar years 2010 through 2015, inclusive, at a nominal rate of 7% per annum convertible monthly. How much is in her account on January 1, 2016?

a. 8968.1
b. 7456.3
c. 9236.8
d. 10,212
e. 5,765.4

Question 22 (3 Points):

At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.

a. 2.75%
b. B. 2.77%
c. C. 2.79%
d. D. 2.81%
e. E. 2.83%

Question 23 (3 Points):

The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to X. The annual effective rate of interest is 9%. Calculate X.

a. 1165
b. 1180
c. 1195
d. 1210
e. 1225

Question 24 (3 Points):

The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to *X*. The annual effective rate of interest is 9%.
Calculate *X*.

a. 1165
b. 1180
c. 1195
d. 1210
e. 1225

Question 25 (3 Points):

At a nominal interest rate of *i* convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year.
Calculate *i*.

a. 2.75%
b. 2.77%
c. 2.79%
d. 2.81%
e. 2.83%