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DEPARTMENT OF MATHEMATICS & STATISTICS
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STAT 319: Probability & Statistics for Engineers & Scientists

Semester 173, Quiz 4

Name: ID #:

Q.No.1: - It is known that in a chemical process the purity of oxygen is related to the percentage hydrocarbons that are present in the main condenser. The purity of the oxygen in % was measured in 20 hydrocarbon levels and the following regression summary information were obtained: $S_{xx} = 0.68$, $S_{xy} = 10.18$, $S_{yy} = 173.37$, where the fitted equation is $\hat{Y} = 75.20 + 14.97X$. Based on this,

a) Compute and interpret the correlation coefficient

b) Compute error variance.

c) Test significance of the regression slope with a significance level of 1%. Use p-value approach.

Q.No.2: - Consider the following MINITAB output:

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The regression equation is
y = 254 + 2.77 x1 - 3.58 x2


Predictor        Coef           SE Coef        T          P
Constant         253.810        4.718          ?          ?
x1               2.7738         0.1846         15.02      ?
x2               -3.5753        0.1526         ?          ?


S = 5.05756   R-Sq = ?   R-Sq(adj) = 98.4%


Analysis of Variance

Source           DF         SS         MS         F
Regression        2         22784      11392      ?
Residual Error    ?         ?          ?
Total            14         23091
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(a) Fill in the missing quantities. You may use the bound for the p-value.

(b) What conclusions can you draw about the significance of regression? The tabulated value from F table is $F_{0.05,2,12} = 3.88$.

(c) What conclusions can you draw about the contribution of the individual predictors to the model?

Q.No.3: - Consider the following partial output from a multiple regression problem:

$$(\boldsymbol{X'X}) = \begin{pmatrix} 7 & 51 & 32 \\ 51 & 471 & 235 \\ 32 & 235 & 163.84 \end{pmatrix}, \quad SS_E = 27.58 \quad \text{and} \quad (\boldsymbol{X'X})^{-\mathbf{1}} = \begin{pmatrix} 1.7996 & -0.06854 & -0.25316 \\ -0.06854 & 0.01008 & -0.00107 \\ -0.25316 & -0.00107 & 0.05708 \end{pmatrix}$$

Use the above to answer the following two problems:

(a) Estimate error variance.

(b) Test the significance of at 5% level of significance. The estimated value for β_1 is 1.4974.

$$S_{XX}=\sum X^2-\frac{\left(\sum X\right)^2}{n},\quad S_{YY}=\sum Y^2-\frac{\left(\sum Y\right)^2}{n},\quad S_{XY}=\sum XY-\frac{\sum X\sum Y}{n}$$

$$Y=\beta_0+\beta_1+\varepsilon,\quad \hat{Y}=\hat{\beta}_0+\hat{\beta}_1X,\quad e=Y-\hat{Y},\quad \hat{\beta}_1=\frac{S_{XY}}{S_{XX}},\quad \hat{\beta}_0=\overline{Y}-\hat{\beta}_1\overline{X}$$

$$SS_T=SS_R+SS_E,\quad SS_T=S_{YY},\quad SS_R=\hat{\beta}_1S_{XY},\quad SS_E=SS_T-SS_R,\quad \hat{\sigma}^2=\frac{SS_E}{n-2}$$

$$r_{XY}=\frac{S_{XY}}{\sqrt{S_{XX}S_{YY}}}=\hat{\beta}_1\sqrt{\frac{S_{XX}}{S_{YY}}}$$ and $$R^2=\frac{SS_R}{SS_T}$$

$\hat{\beta}_0 \pm T_{\frac{\alpha}{2},v}\sqrt{\hat{\sigma}^2\left[\frac{1}{n}+\frac{\overline{X}^2}{S_{XX}}\right]}$	$T=\frac{\hat{\beta}_0-\beta_{00}}{\sqrt{\hat{\sigma}^2\left[\frac{1}{n}+\frac{\overline{X}^2}{S_{XX}}\right]}},\ v=n-2$
$\hat{\beta}_1 \pm T_{\frac{\alpha}{2},v}\sqrt{\frac{\hat{\sigma}^2}{S_{XX}}}$	$T=\frac{\hat{\beta}_1-\beta_{10}}{\sqrt{\frac{\hat{\sigma}^2}{S_{XX}}}},\ v=n-2$
$\hat{y}_0 \pm T_{\frac{\alpha}{2},v}\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{\left(x_0-\overline{X}\right)^2}{S_{XX}}\right]},\quad v=n-2$	
$\hat{\mu}_{Y\mid X=x_0} \pm T_{\frac{\alpha}{2},v}\sqrt{\hat{\sigma}^2\left[\frac{1}{n}+\frac{\left(x_0-\overline{X}\right)^2}{S_{XX}}\right]},\quad v=n-2$	

$$y=X\beta+\varepsilon,\quad \hat{Y}=X\hat{\beta},\quad e=Y-\hat{Y},\quad \hat{\beta}=\left(X'X\right)^{-1}X'Y,\quad s.e.\left(\hat{\beta}_j\right)=\sqrt{\hat{\sigma}^2C_{jj}}$$

$$SS_T=SS_R+SS_E,\quad SS_T=Y'Y-\frac{\left(\sum Y\right)^2}{n},\ Y'Y=\sum Y^2,\quad SS_R=\hat{\beta}'X'Y-\frac{\left(\sum Y\right)^2}{n},$$

$$SS_E=SS_T-SS_R\quad \hat{\sigma}^2=\frac{SS_E}{n-p},\quad R^2=\frac{SS_R}{SS_T}$$ and $$R^2_{ajd}=1-\frac{SS_E/(n-p)}{SS_T/(n-1)}$$

$$F=\frac{SS_R/k}{SS_E/(n-p)},\quad v_1=k$$ and $$v_2=n-p$$

$$T=\frac{\hat{\beta}_j-\beta_{j0}}{s.e.\left(\hat{\beta}_j\right)},\quad v=n-p \qquad \hat{\beta}_j \pm T_{\frac{\alpha}{2},v}\,s.e.\left(\hat{\beta}_j\right)$$

$$\hat{y}_0 \pm T_{\frac{\alpha}{2},v}\sqrt{\hat{\sigma}^2\left[1+x_0'\left(X'X\right)^{-1}x_0\right]},\quad v=n-p$$

$$\hat{\mu}_{Y\mid X=x_0} \pm T_{\frac{\alpha}{2},v}\sqrt{\hat{\sigma}^2\left[x_0'\left(X'X\right)^{-1}x_0\right]},\quad v=n-p$$