King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

Math 371 Final Exam

Summer Semester 2018 (173)

Time Allowed: 180 Minutes

Name:_____ ID#:_____ Sec.

- Mobiles are not allowed in this exam.
- Write all steps clear.

Question #	Answer
1/7	
2/8	
3/10	
4/15	
5/10	
6/10	
7/10	
8/10	
9/10	
10/10	
11/10	
12/10	
Total	
	Grand Total out of 120

Q:1(7 points) The fixed points of $x = cosx$ in the interval $[0, \pi/2]$ with initial guess $x_o = 1$ and Tolerance = 10^{-1} is approximately equal to:

Q:2(8 points) Approximate value of $\int_0^1 xe^{sinx}\, dx\, using\ composite\ Simpson's$ rule with n=4

Q:3 (10 points) Given $f(x) = e^{2x},\ 1 \leq x \leq 2$

a) Analyze the round-off errors for the formula

$$f''(x) = \frac{1}{h^2}[f(x_0 - h) - 2f(x_0) + f(x_0 + h)] - \frac{h^2}{12}f^{(4)}(\xi)$$

b) If the values of f are given in 4 decimal places, find the **optimal *h***.

Q:4 (5+10 points)

a) Show that the initial-value problem has a unique solution

$$y' = 2y\,cost, \qquad 0 \le t \le 2, \;\; y(0) = 2$$

b) Use Runge-Kutta of order four to approximate the solution in part(a) (find $w_2\ with\ h = 0.5$)

Q:5 (10 points) Use Gaussian elimination with scaled partial pivoting and three-digit rounding to solve the system

$$
\begin{aligned}
2.11x - 4.21y + 0.921z &= 2.01, \\
4.01x + 10.2y - 1.12z &= -3.09, \\
1.09x + 0.987y + .832z &= 4.21
\end{aligned}
$$

Q:6 (10 points) Using the data points, (0, 0), (1, -1), (3, 3) second order interpolating polynomial using Newton's divided difference interpolation,

Q:7 (10 points) Using Gauss–Seidel iterative method to solve the linear system

$$3x - y + z = 1$$

$$3x + 6y + 2z = 0$$

$$3x + 3y + 7z = 4$$

with initial guess $\bar{x} = (0, 0, 0)$, find two iterations.

Q:8 (10 points) The following data are exponentially related $y = be^{ax}$

$(1, 5.1), (1.25, 5.79), (1.5, 6.53), (1.75, 7.45), (2, 8.46).$

By least squares approximation find a and b.

Q:9(10 points) The boundary-value problem

$$y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}, \quad 0 \le x \le 1, \qquad y(0) = -1, y(1) = 0, h = 0.2$$

By using Linear Finite-Difference method write the problem in matrix form.

Q:10(10 points)
Write a MATLAB code that does the following:

a) Determine the linear least square polynomial for a set of data of the form

x	1	2	3	4	5
y	1.3	3.5	4.2	5	7

b) Plot the set of data and its linear fit in the same figure window.

Q:11 (10 points) Write a MATLAB code to approximate the solution of the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \qquad y(0) = 0.5$$

using Euler method with h = 0.2.

Also include the commands to plot the numerical solution and the exact solution

$$y = 1 + t^2 + 2t - 0.5e^t$$

Q:12(10 points) Write a MATLAB code to approximate the solution of the equation

$$\ln(x-1)+\cos(x-1)=0, 1.3 \leq x \leq 2,$$

By using Newton's method with accurate 0.001.

The End