King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE MASTER

Math 202
Final Exam
Term 173
Wednesday August 15, 2018
Time Allowed: 180 minutes

CODE MASTER

Name: ______________________________

ID: ____________________ Sec: ____________________.

Check that there is total of 14 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The implicit solution of the initial-value problem

$$2\sin 2x\,dx + y\cos^3 2x\,dy = 0, \qquad y(0) = 2,$$

is equal to:

a) $y^2 = 4 - \tan^2 2x$

b) $y^2 = 4 + \tan^2 2x$

c) $y^2 = 2 + 2\sec^2 2x$

d) $y^2 = 4 + \tan 2x$

e) $y^2 = 4 - 2\tan 2x$

2. If the differential equation

$$\left(\frac{1}{3}x^3 + \ln x + x \sec y \, \tan y + x\right) dy + \left(x^2 y + y/x + \boldsymbol{S}(\boldsymbol{y})\right) dx = 0$$

is **EXACT**, then $\boldsymbol{S}(\boldsymbol{0}) =$

a) 1

b) 0

c) $\frac{1}{2}$

d) $\ln 2$

e) 2

3. The implicit solution of the differential equation

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)\,y}$$

is given by:

a) $(1+x^2)^2\, y^2 - 2x = C$

b) $(1+x^2)^2\, y^2 + 2x = C$

c) $y^2 - 2x\,(1+x^2) = C$

d) $y^2 + 2x\,(1+x^2) = C$

e) $y^2 - 2x\,(1+x^2)^2 = C$

4. The population of a country is assumed to increase at a rate proportional to the number of people present at time t. If the initial population is 1 million and this number is doubled in 2 years, it will take the population to reach 4 million in:

a) 4 years

b) 5 years

c) $\ln 4$ years

d) $\ln 8$ years

e) $5 \ln 2$ years

5. The function $y_1 = x^2$ is a solution of the differential equation

$$(x^2 - 3x + 2)\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + 4\frac{x - 1}{x^2}y = 0, \qquad x > 2.$$

The method of **Reduction of Order** produces the second solution $y_2 =$

a) $\mathbf{\frac{4-3x}{6x}}$

b) $\frac{3x-4}{6x}$

c) $\frac{3-4x}{6x^2}$

d) $\frac{4-3x}{6x^2}$

e) $\frac{4}{3x^2}$

6. The solution $\boldsymbol{y}(\boldsymbol{x})$ of the third order initial-value problem

$$\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0, \qquad y(0)=0,\ \ y'(0)=0,\ \ y''(0)=1,$$

satisfies $\boldsymbol{y}(\ln 3) =$

a) $\mathbf{\frac{10}{27}}$

b) $\frac{1}{27}$

c) $\frac{4}{27}$

d) $\frac{1}{6}$

e) 0

7. If $D^4 + a\,D^3 + b\,D^2 + c\,D + 2$ annihilates the function

$$\frac{3x - \cos x}{e^x}$$

then the value of $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ is equal to:

a) 17

b) 15

c) 16

d) 18

e) 19

8. Let $\boldsymbol{y_p(x)}$ be the particular solution of the Cauchy-Euler differential equation

$$x^2y'' - 4xy' + 6y = 2x^4 + x^2, \qquad x > 0.$$

Then $\boldsymbol{y_p(e^{-1})} =$

a) $\boldsymbol{e^{-4}}$

b) e^{-2}

c) $e^4 + 2e^2$

d) $e^2 - 2e^4$

e) $e^4 - 2e^2$

9. The second order differential equation

$$y'' - 3xy' - 3y = 0,$$

possesses **two** linearly independent power series solutions $\boldsymbol{y_1(x)}$ and $\boldsymbol{y_2(x)}$ about the ordinary point $x_0 = 0$. Then the sum of the **FIRST THREE NONZERO TERMS** in $\boldsymbol{y_1}$ and in $\boldsymbol{y_2}$ at $\boldsymbol{x = 1}$ are:

a) $\frac{29}{8}$ and $\frac{13}{5}$

b) $\frac{5}{2}$ and $\frac{11}{5}$

c) $\frac{29}{8}$ and $\frac{3}{5}$

d) $\frac{17}{5}$ and 2

e) $\frac{21}{5}$ and $\frac{13}{5}$

10. $\boldsymbol{x_0 = 0}$ is a regular singular point of the differential equation

$$3xy'' + 2xy' - 2y = 0$$

Using the Frobenius method, the **SUM OF THE FIRST THREE TERMS** of the solution $\boldsymbol{y(x)}$ at $\boldsymbol{x = 3}$ corresponding to the largest root of the indicial equation equals:

a) 3

b) 0

c) $\frac{1}{2}$

d) $\frac{9}{16}$

e) $\frac{27}{32}$

11. The general solution of the first order homogenous system

$$X' = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 4 & 3 & 1 \end{pmatrix} X$$

is given by

$$X = c_1 \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} e^{\lambda_1 t} + c_2 \begin{pmatrix} c \\ d \\ 1 \end{pmatrix} e^{\lambda_2 t} + c_3 \begin{pmatrix} e \\ f \\ 16 \end{pmatrix} e^{-3t}, \qquad \lambda_1 \neq \lambda_2$$

Then $\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{c} \cdot \boldsymbol{d} + \boldsymbol{e} \cdot \boldsymbol{f} =$

a) 78

b) 75

c) 81

d) 84

e) 87

12. Consider the initial-value problem

$$X' = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix} X, \qquad X(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

One eigenvector of this system corresponding to an incomplete eigenvalue $\boldsymbol{\lambda}$ is $K_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. Then the solution of the IVP is $\boldsymbol{X}(\boldsymbol{t}) =$

a) $\boldsymbol{\begin{pmatrix} 6t+1 \\ 2t \end{pmatrix} e^{-3t}}$

b) $\begin{pmatrix} 2t+1 \\ 3t \end{pmatrix} e^{-3t}$

c) $\begin{pmatrix} 2t+1 \\ 0 \end{pmatrix} e^{-3t}$

d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-3t}$

e) $\begin{pmatrix} t+1 \\ t \end{pmatrix} e^{-3t}$

13. Consider the nonhomogeneous system

$$X' = A\,X + \begin{pmatrix} \sin 2t \\ \cos 2t \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1\,e^t \begin{pmatrix} \sin 2t \\ \cos 2t \end{pmatrix} + c_2\,e^t \begin{pmatrix} -\cos 2t \\ \sin 2t \end{pmatrix},$$

then the **particular solution, $\boldsymbol{X_p(t)}$,** of the nonhomogeneous system **AT $\boldsymbol{t = 0}$** equals:

a) $\begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \end{pmatrix}$

b) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

c) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

e) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

14. If the Matrix Exponential of

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix}$$

is given by

$$e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ h_1(t) & 1 & 0 \\ h_2(t) & h_3(t) & 1 \end{pmatrix},$$

then $\boldsymbol{h_1(1) + h_2(2) + h_3(3)} =$

a) 22

b) 17

c) 19

d) 20

e) 23