Math201.01 & 2, Quiz #2, Term 173

Name: Solutions

ID#: **Serial #:**

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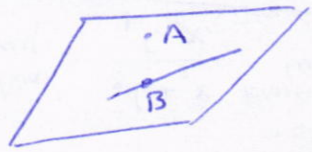
1. **[2.5 points]** Find an equation of the plane that contains the point $(1,5,-2)$ and the line $x = 3 - 3t, y = 1 + t, z = 2 - t$.
2. **[2.5 points]** Identify (name, vertex, axis) and sketch the surface $3z^2 + 2y^2 - 4y - x = -2$.
3. **[2.5 points]** Find and sketch the domain of $f(x,y) = \frac{\ln(1-y)}{\sqrt{y-x^2}}$.
4. **[2.5 points]** Find the limit if it exists: $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$. a point on the plane : $A(1,5,-2)$

. a point on the line : $B(3,1,2)$, when $t=0$.

. a vector parallel to the line : $\vec{v} = \langle -3, 1, -1\rangle$

. $\vec{BA} = \langle -2, 4, -4\rangle$

. a normal vector to the plane is $\vec{n} = \vec{BA} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & -4 \\ -3 & 1 & -1 \end{vmatrix} = \langle 0, 10, 10\rangle$

. An equation for the plane is

$$0\cdot(x-1) + 10(y-5) + 10(z+2) = 0$$

$$\Rightarrow \quad y + z = 3$$

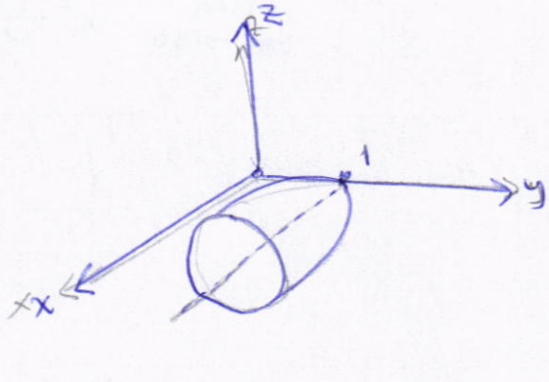
$\boxed{2}$ $3z^2 + 2(y^2 - 2y + 1) = x - 2 + 2$

$x = 2(y-1)^2 + 3z^2$

name: an elliptic paraboloid

vertex: $(0,1,0)$

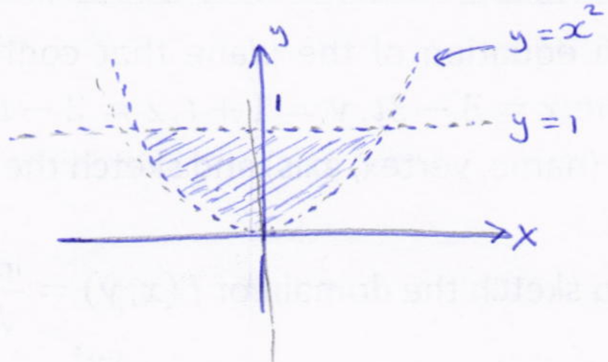
axis: the line // x-axis & through $(0,1,0)$.



$\boxed{3}$ $f(x,y) = \dfrac{\ln(1-y)}{\sqrt{y-x^2}}$

Domain $= \{(x,y) \in \mathbb{R}^2 : 1-y>0 \text{ and } y-x^2>0\}$

$= \{(x,y) \in \mathbb{R}^2 : y<1 \text{ and } y>x^2\}$



$\boxed{4}$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x y^4}{x^2+y^8}$

- along the x-axis: $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x y^4}{x^2+y^8} = \lim_{(x,y)\to(0,0)} \left.\frac{x y^4}{x^2+y^8}\right|_{y=0} = \lim_{(x,y)\to(0,0)} \frac{x\cdot 0}{x^2+0} = \lim_{(x,y)\to(0,0)} 0 = 0$$

- along the Curve $x=y^4$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=y^4}} \frac{xy^4}{x^2+y^8} = \lim_{(x,y)\to(0,0)} \left.\frac{xy^4}{x^2+y^8}\right|_{x=y^4} = \lim_{y\to0} \frac{y^4\cdot y^4}{y^8+y^8} = \lim_{y\to0} \frac{1}{2} = \frac{1}{2}$$

Since the limits along the above two paths are not equal, then

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$ DNE.