Math201.01, Quiz # 4, Term 173

Name:

Solutions

ID #: Serial #:

1. **[3 points]** Set up a double integral for the volume of the solid below the paraboloid $z + x^2 + 5y^2 = 81$ and above the region in the $xy-plane$ bounded by the curves $x = 2y^2$ and $x = 6$. **DO NOT EVALUATE THE INTEGRAL.**
2. **[4 points]** Evaluate the double integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy.$$

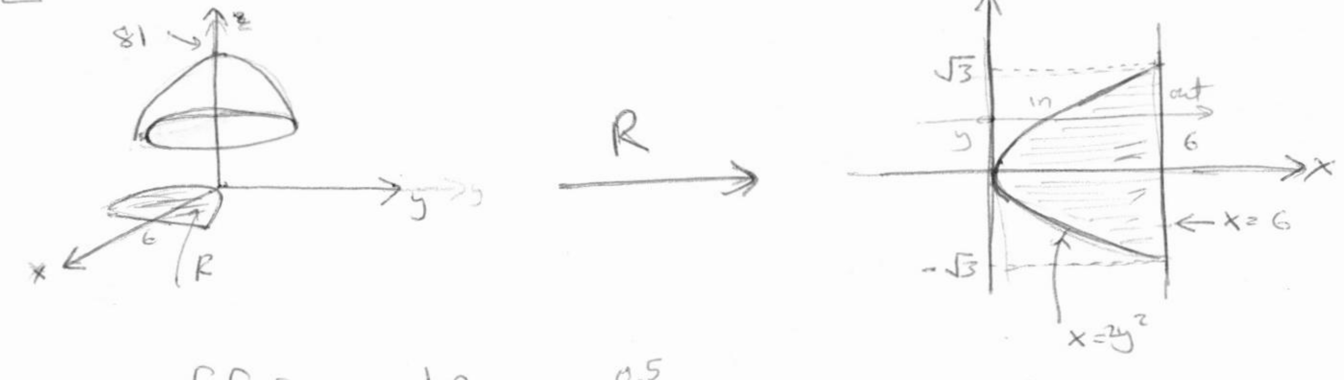
3. **[3 points]** Evaluate $\iint_R \sin(x^2 + y^2)\, dA$, where $R = \{(x, y): 1 \le x^2 + y^2 \le 6, x \le 0\}$.

Good luck,

Ibrahim Al-Rasasi

$2y^2 = 6 \Rightarrow y^2 = 3$
$\Rightarrow y = \pm\sqrt{3}$

$\boxed{1}$ $z = 81 - x^2 - 5y^2$



$V = \iint_R f(x,y)\, dA$ 0.5

$= \iint_R (81 - x^2 - 5y^2)\, dA$ 0.5

$= \int_{-\sqrt{3}}^{\sqrt{3}} \int_{2y^2}^{6} (81 - x^2 - 5y^2)\, dx\, dy$

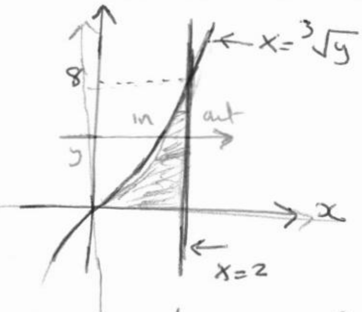
1 1

R is a region of Type II

$R: 2y^2 \le x \le 6$
$-\sqrt{3} \le y \le \sqrt{3}$

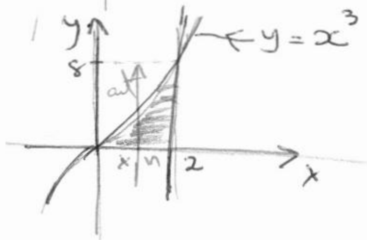
②

[2] $\int_0^8 \int_{\sqrt[3]{y}}^{2} e^{x^4}\, dx\, dy$, Not easy to Integrate



R : $\sqrt[3]{y} \le x \le 2,\ 0 \le y \le 8$, Type II

R : $0 \le x \le 2,\ 0 \le y \le x^3$, Type I



$$\cdot \int_0^8 \int_{\sqrt[3]{y}}^{2} e^{x^4}\, dx\, dy = \int_0^2 \int_0^{x^3} e^{x^4}\, dy\, dx \quad \underline{\underline{2}}$$

$$= \int_0^2 e^{x^4} \cdot y \Big]_{y=0}^{y=x^3} dx \quad \underline{0.5}$$

$$= \int_0^2 e^{x^4} \cdot x^3\, dx \quad \underline{0.5}$$

$$= \frac{1}{4} e^{x^4} \Big]_0^2 \quad \underline{0.5}$$

$$= \frac{1}{4} \left(e^{16} - 1\right) \quad \underline{0.5}$$

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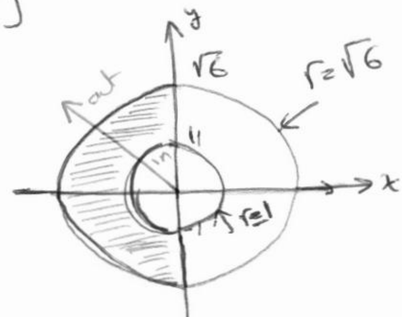
[3] $\iint_R \sin(x^2+y^2)\, dA$, $R = \{(x,y) : 1 \le x^2+y^2 \le 6,\ x \le 0\}$

$\cdot$ Change to polar Coord.:



R : $1 \le r \le \sqrt{6}$
$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$$\iint_R \sin(x^2+y^2)\, dA = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_1^{\sqrt{6}} \sin(r^2) \cdot r\, dr\, d\theta \quad \underline{\underline{2}}$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{1}{2} \cos(r^2) \Big]_{r=1}^{r=\sqrt{6}} d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{1}{2} \left(\cos 6 - \cos 1\right) d\theta \quad \underline{0.5}$$

$$= \frac{1}{2}(\cos 1 - \cos 6) \cdot \theta \Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \frac{1}{2}(\cos 1 - \cos 6) \cdot \pi$$

$$= \frac{\pi}{2}(\cos 1 - \cos 6). \quad \underline{0.5}$$

Math201.02, Quiz #4, Term 173

Name:

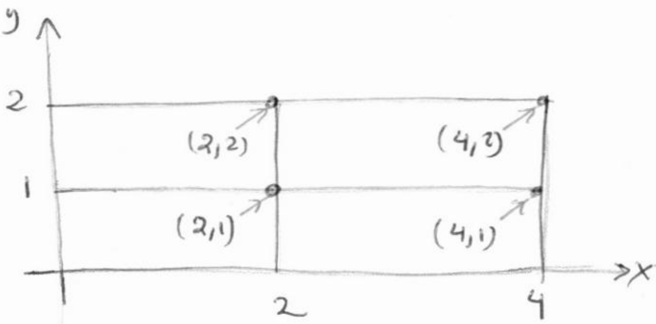
ID #: Solutions

Serial #:

1. **[3 points]** Let $R = [0,4] \times [0,2]$. Estimate the double integral $\iint_R \ln(x+y)\, dA$ using double Riemann sums with $m = 2$ and $n = 2$ and taking as sample points the **upper right corner** of each sub-rectangle.
2. **[4 points]** Evaluate $\iint_R \sin\left(\frac{x}{y}\right) dA$, where R is the region bounded by the curves $y = x, y = 1, x = 0$.
3. **[3 points]** Find the average value of $f(x,y) = xy$ over the region $R = \{(x,y): x^2 + y^2 \leq 4, y \leq 0\}$.

Good luck,

Ibrahim Al-Rasasi



[1] $\Delta x = \frac{4-0}{2} = 2$

$\Delta y = \frac{2-0}{2} = 1$ 0.5

$\Delta A = \Delta x \, \Delta y = (2)(1) = 2$

$f(x,y) = \ln(x+y)$.

$\iint_R f(x,y)\, dA \approx f(2,1)\cdot\Delta A + f(2,2)\cdot\Delta A + f(4,1)\cdot\Delta A + f(4,2)\cdot\Delta A$ 2

$= \ln 3 \cdot 2 + \ln 4 \cdot 2 + \ln 5 \cdot 2 + \ln 6 \cdot 2$

$= 2(\ln 3 + \ln 4 + \ln 5 + \ln 6)$ 0.5

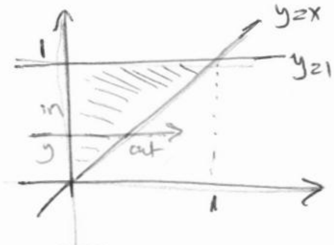
$\overset{\text{or}}{=} 2\ln(3\cdot4\cdot5\cdot6)$

$= 2\ln(360)$

2 $\iint\limits_R \sin\left(\frac{x}{y}\right) dA$

. It is easier to integrate first w.r.t x.

So we choose $dA = dx\,dy$ (R is Type II)



Type II

$R: \ 0 \le x \le y$

$0 \le y \le 1$

. $\iint\limits_R \sin\left(\frac{x}{y}\right) dA = \int_0^1 \int_0^y \sin\left(\frac{x}{y}\right) dx\,dy$ 2

$= \int_0^1 \left. -y\cos\left(\frac{x}{y}\right)\right]_{x=0}^{x=y} dy$ 0.5

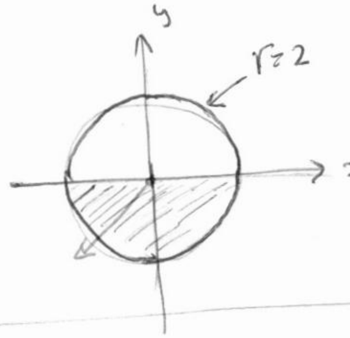
$= \int_0^1 -y(\cos 1 - 1)\,dy$ 0.5

$= (1-\cos 1)\int_0^1 y\,dy$

$= (1-\cos 1)\cdot \left.\frac{1}{2}y^2\right]_0^1$ 0.5

$= \frac{1-\cos 1}{2}$ 0.5

3 $f(x,y) = xy \ , \quad R = \{(x,y): x^2+y^2 \le 4 \ , y \le 0\}$



. Use polar Coord.

. $f_{ave} = \frac{1}{\text{area}(R)} \iint\limits_R f(x,y)\,dA$

$R: \ 0 \le r \le 2$

$\pi \le \theta \le 2\pi$

$\text{area}(R) = \frac{1}{2}\pi(2)^2$

$= 2\pi$

2

$= \frac{1}{2\pi}\int_\pi^{2\pi}\int_0^2 (r\cos\theta)(r\sin\theta)\cdot r\,dr\,d\theta$

$= \frac{1}{2\pi}\int_\pi^{2\pi}\int_0^2 r^3 \sin\theta\cos\theta\,dr\,d\theta$

$= \frac{1}{2\pi}\int_\pi^{2\pi} \sin\theta\cos\theta \cdot \underbrace{\left.\frac{1}{4}r^4\right]_{r=0}^{r=2}}_{=4} d\theta$

$= \frac{2}{\pi}\int_\pi^{2\pi} \sin\theta\cos\theta\,d\theta$ 0.5 $\boxed{u = \sin\theta}$

$= \frac{2}{\pi} \cdot \left.\frac{\sin^2\theta}{2}\right]_\pi^{2\pi}$

$= \frac{2}{\pi}(0-0)$

$= 0$ 0.5