Math201.01, Quiz #1, Term 173

Name: Solutions

ID#: **Serial #:**

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

$$x = \sqrt{t+2}, \qquad y = -\sqrt{t-1}, \qquad t \geq 1.$$

2. **[3 points]** Find the length of the parametric curve

$$x = e^t + e^{-t}, y = 5 - 2t, 0 \leq t \leq 3.$$

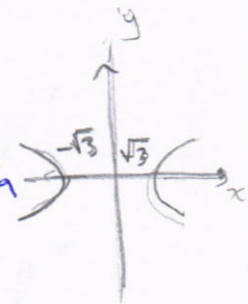
3. **[4 points]** Let R be the region inside the circle $r = 4\sin(\theta)$ and above the polar curve $r = 3csc\theta$. Sketch the region R and find its area.

Good luck,

Ibrahim Al-Rasasi

1. $x = \sqrt{t+2} \Rightarrow x^2 = t+2 \Rightarrow t = x^2 - 2$
$y = -\sqrt{t-1} \Rightarrow y^2 = t-1 \Rightarrow t = y^2+1$ $\Big\} \Rightarrow x^2 - 2 = y^2 + 1$

$\Rightarrow x^2 - y^2 = 3$, a hyperbola



1.5

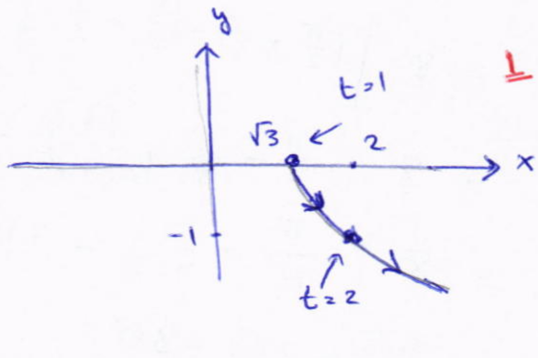
- For $t \geq 1$, $x = \sqrt{t+2} \geq 0$ and $y = -\sqrt{t-1} \leq 0$. So we take the part of the hyperbola in the 3rd quadrant

- For directions

$t=1 \Rightarrow (x,y) = (\sqrt{3}, 0)$, the initial pt

0.5

$t=2 \Rightarrow (x,y) = (2,-1)$



1

$\boxed{x^2 - y^2 = 3, \quad x \geq \sqrt{3}, \quad y \leq 0}$

②

[2]

$$L = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\; dt \qquad \underline{\underline{0.5}}$$

- $x = e^t + e^{-t} \Rightarrow \frac{dx}{dt} = e^t - e^{-t}$ $\quad \underline{0.5}$
- $y = 5 - 2t \Rightarrow \frac{dy}{dt} = -2$
- $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{2t} - 2 + e^{-2t} + 4$ $\quad \underline{0.5}$

$$= e^{2t} + 2 + e^{-2t} \qquad \underline{0.5}$$

$$= (e^t + e^{-t})^2 \qquad \underline{0.5}$$

$$= \int_0^3 e^t + e^{-t}\; dt$$

$$= e^t - e^{-t}\Big]_0^3 \qquad \underline{0.5}$$

$$= (e^3 - e^{-3}) - (1 - 1)$$

$$= e^3 - e^{-3}$$

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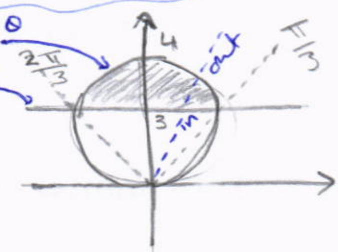
[3]

- $r = 4\sin\theta$
- $r = 3\csc\theta = \frac{3}{\sin\theta} \Rightarrow r\sin\theta = 3 \Rightarrow y = 3$



$\underline{1}$

- points of intersection:

$4\sin\theta = 3\csc\theta \Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3} \in QI, II$ $\quad \underline{\underline{0.5}}$

- By Symmetry about the y-axis,

$$A = 2 \cdot \int_{\pi/3}^{\pi/2} \tfrac{1}{2}\left[(4\sin\theta)^2 - (3\csc\theta)^2\right] d\theta \qquad \underline{1}$$

$$= \int_{\pi/3}^{\pi/2} 16 \cdot \frac{1 - \cos(2\theta)}{2} - 9\csc^2\theta \; d\theta \qquad \underline{0.5}$$

$$= \int_{\pi/3}^{\pi/2} 8\,(1 - \cos(2\theta)) - 9\csc^2\theta \; d\theta$$

$$= 8\left(\theta - \tfrac{1}{2}\sin(2\theta)\right)\Big]_{\pi/3}^{\pi/2} + 9\cot\theta\Big]_{\pi/3}^{\pi/2} \qquad \underline{0.5}$$

$$= 8\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{3} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)\right] + 9\left(0 - \frac{1}{\sqrt{3}}\right)$$

$$= 8\left(\frac{\pi}{2} - \frac{\pi}{3} + \frac{\sqrt{3}}{4}\right) - \frac{9}{\sqrt{3}}$$

$$= 8\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right) - 3\sqrt{3}$$

$$= \frac{4\pi}{3} + 2\sqrt{3} - 3\sqrt{3}$$

$$= \frac{4\pi}{3} - \sqrt{3} \qquad \underline{0.5}$$

Math201.02, Quiz #1, Term 173

Name: Solutions

ID#: **Serial #:**

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

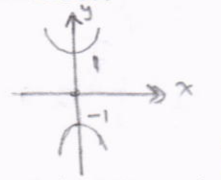
$$x = 1 + tant, \qquad y = sect, \qquad -\pi/2 < t < \pi/2.$$

2. **[3 points]** Find the slope of the tangent line to the polar curve $r = 2 + \cos(4\theta)$ at the point corresponding to $\theta = \pi/4$.
3. **[4 points]** Let R be the region in **the first quadrant** that lies inside the cardioid $r = 1 + cos\theta$ and outside the cardioid $r = 1 + sin\theta$. Sketch the region R and find its area.

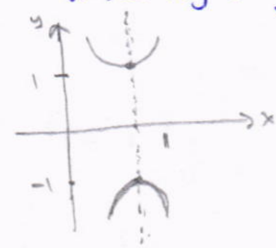
Good luck,

Ibrahim Al-Rasasi

- $y^2 - x^2 = 1$



- $y^2 - (x-1)^2 = 1$, shift 1 unit to the right



[1] • Since $1 + \tan^2 t = \sec^2 t$, then $1 + (x-1)^2 = y^2$

or $y^2 - (x-1)^2 = 1$, a hyperbola 1.5

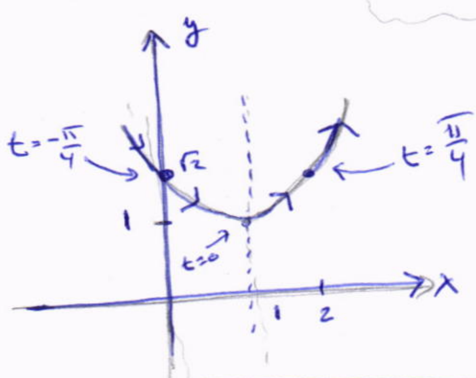
• Since $y = \sec t > 0$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$,

we take the upper branch of the hyperbola

• For directions

t	(x, y)
$-\frac{\pi}{4}$	$(0, \sqrt{2})$
0	$(1, 1)$
$\frac{\pi}{4}$	$(2, \sqrt{2})$

0.5



1

$$\boxed{y^2 - (x-1)^2 = 1, \quad y \geq 1.}$$

2| $r = 2+\cos(4\theta)$, $\theta = \frac{\pi}{4}$, $f(\theta) = 2+\cos(4\theta)$

- $$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} \quad 1 \quad = \frac{-4\sin(4\theta)\cdot\sin\theta + (2+\cos(4\theta))\cdot\cos\theta}{-4\sin(4\theta)\cdot\cos\theta - (2+\cos(4\theta))\cdot\sin\theta} \quad 1$$

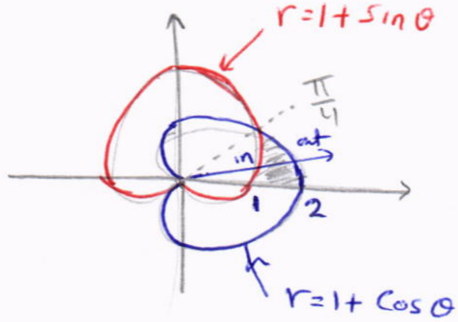
- $$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{0 + (2-1)\,1/\sqrt{2}}{0 - (2-1)\,1/\sqrt{2}} = -1$$

0.5 0.5

3| • pts of intersection

$1+\cos\theta = 1+\sin\theta \implies \cos\theta = \sin\theta$

$\implies \theta = \frac{\pi}{4} \in QI$ 0.5



1

$$A = \int_0^{\frac{\pi}{4}} \frac{1}{2}\left[(1+\cos\theta)^2 - (1+\sin\theta)^2\right] d\theta \quad 1$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} 1+2\cos\theta + \cos^2\theta - 1 - 2\sin\theta - \sin^2\theta \; d\theta$$

" $\cos^2\theta - \sin^2\theta = \cos(2\theta)$ "

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} 2\cos\theta - 2\sin\theta + \cos(2\theta) \; d\theta$$

1

$$= \frac{1}{2}\left[2\sin\theta + 2\cos\theta + \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left[\left(\sqrt{2}+\sqrt{2}+\frac{1}{2}\right) - (0+2+0)\right]$$

$$= \frac{1}{2}\left(2\sqrt{2} - \frac{3}{2}\right)$$

$$= \sqrt{2} - \frac{3}{4}$$

0.5