King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 102

EXAM I

173

Monday 9/7/2018

Net Time Allowed: 120 minutes

MASTER VERSION

1. Using four approximating rectangles and left endpoints. The area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \pi$ is approximately equals to

(a) $\frac{\pi}{4}(1+\sqrt{2})$

(b) $\frac{\pi}{4}$

(c) $\frac{\sqrt{2}}{2}\pi$

(d) $\frac{\pi}{4\sqrt{2}}$

(e) $\frac{\pi}{4}\left(1+\frac{1}{\sqrt{2}}\right)$

2. $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)\,dx =$

(a) $3+\frac{9}{4}\pi$

(b) $\frac{9}{4}\pi$

(c) 3π

(d) $1+\frac{3\pi}{2}$

(e) $9\pi-3$

3. If $f(x) = \int_0^x (1-t^2)\, e^{t^2}\, dt$ for $-\infty < x < \infty$, then f is decreasing on

(a) $(-\infty, -1) \cup (1, \infty)$

(b) $(-1, 1)$

(c) $(-\infty, +1)$

(d) $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$

(e) $(-1, \infty)$

4. If $g(x) = \int_{\cos x}^{\sin x} \ln(2+3t)\, dt$, then $g'(0) =$

(a) $\ln 2$

(b) $\ln 3$

(c) $\ln 5$

(d) $\ln 6$

(e) 0

5. $\displaystyle\int_1^2 \frac{\sin\left(\frac{\pi}{x}\right)}{x^2}\,dx =$

(a) $1/\pi$

(b) 1

(c) $-1/2$

(d) $-\pi$

(e) $\pi/2$

6. $\displaystyle\lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{1}{n}\right)\frac{1}{\sqrt{1-\left(\frac{i}{2n}\right)^2}}\right] =$

(a) $\displaystyle\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx$

(b) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{1-x^2}\,dx$

(c) $\displaystyle\int_{-\frac{1}{2}}^{0} \frac{1}{\sqrt{1-x^2}}\,dx$

(d) $\displaystyle\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \frac{2}{\sqrt{1-2x^2}}\,dx$

(e) $\displaystyle\int_{-2}^{2} \frac{2}{\sqrt{x}}\,dx$

7. Suppose that f and g are continuous functions on $[a, b]$ and a, b, e, m, n are constants. Which one of the following is always **True**

(a) $\int_{-2}^{2}(ex^2 + mx + n)\,dx = 2\int_{0}^{2}(ex^2 + n)\,dx$

(b) $\int_{a}^{b}(f(x)\cdot g(x))\,dx = \left(\int_{a}^{b} f(x)\,dx\right)\cdot\left(\int_{a}^{b} g(x)\,dx\right)$

(c) $\int_{a}^{b} x\,f(x)\,dx = x\int_{a}^{b} f(x)\,dx$

(d) $\int_{a}^{b} \sqrt{f(x)}\,dx = \sqrt{\int_{a}^{b} f(x)\,dx}$

(e) If $\int_{0}^{1} f(x)\,dx = 0$, then $f(x) = 0$ for $0 \le x \le 1$

8. Suppose each of the regions A, B, and C bounded by the graph of f and the $x-$ axis has area 1, as shown in the figure. The value of $\int_{-2}^{1}(3\,f(x) + 2x + 6)\,dx =$

(a) 12

(b) 6

(c) 3

(d) 15

(e) 1

9. Expressing the limit as a definite integral,

$$\lim_{n\to\infty}\left[\frac{1}{n}\left(\sqrt{\frac{2}{n}}+\sqrt{\frac{4}{n}}+\sqrt{\frac{6}{n}}+\ldots+\sqrt{2}\right)\right]=$$

(a) $\frac{2\sqrt{2}}{3}$

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{2}}{3}$

(d) $\frac{2}{3}$

(e) $\frac{1}{3}$

10. A partical is moving in a straight line with velocity $v(t) = 2\ \sin\ 2t$ (m/sec). Then the total distance covered in meters by the partical in the time interval $\left[0, \frac{3\pi}{4}\right]$ is

(a) 3

(b) 4

(c) $1-\frac{1}{\sqrt{2}}$

(d) $\frac{1}{\sqrt{2}}$

(e) $1+\frac{1}{\sqrt{2}}$

11. $\displaystyle\int_{\pi/4}^{\pi/3} \frac{2\sec^2 x \ln(\tan x)}{\tan x}\, dx =$

(a) $\left(\dfrac{\ln 3}{2}\right)^2$

(b) $\ln 3$

(c) $\left(\dfrac{\ln 5}{2}\right)^2$

(d) $\left(\dfrac{\ln 6}{2}\right)^2$

(e) $\ln 6$

12. $\displaystyle\int_{0}^{2/5} \frac{d\omega}{4+25\,\omega^2} =$

(a) $\dfrac{\pi}{40}$

(b) $\dfrac{3\pi}{5}$

(c) $\dfrac{2\pi}{5}$

(d) $\dfrac{\pi}{5}$

(e) $\dfrac{3\pi}{40}$

13. The area of the region bounded by the curves $y+x^2=6$ and $y+2x-3=0$ is equal to

(a) $\frac{32}{3}$

(b) 16

(c) 11

(d) $\frac{35}{2}$

(e) $\frac{16}{3}$

14. The volume of the solid generated by rotating the region bounded by $y=\sqrt{\cos\left(\frac{\pi\, x}{4}\right)}$, $x=1, x=2$, and $y=0$ about the x-axis is equal to

(a) $4-2\sqrt{2}$

(b) $\frac{2-\sqrt{2}}{4}$

(c) $2\pi+\sqrt{2}$

(d) $4\,\pi$

(e) $4\,\pi-\sqrt{2}$

15. $\displaystyle\int_0^{1/\sqrt{2}} \frac{x\,dx}{\sqrt{1-x^4}} =$

(a) $\pi/12$

(b) $\pi/3$

(c) 1

(d) $\pi/4$

(e) π

16. The area of the region bounded by the curves $y = 2+|x-1|$, and $y = \frac{1}{2}x+3$ is given by the integral

(a) $\displaystyle\int_0^1 \left(\frac{3}{2}x\right)\,dx + \int_1^4 \left(2-\frac{1}{2}x\right)\,dx$

(b) $\displaystyle\int_1^4 (x+2)\,dx$

(c) $\displaystyle\int_0^1 x^2\,dx + \int_1^4 (2-x)\,dx$

(d) $\displaystyle\int_1^4 \left(\frac{3}{2}x+4\right)\,dx$

(e) $\displaystyle\int_{-1}^2 \left(\frac{1}{2}x-\frac{1}{3}\right)\,dx + \int_2^4 \left(3-\frac{3}{2}x\right)\,dx$

17. $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\sin^9 x + 1}{\cos^2 x}\, dx =$

(a) 2

(b) $\dfrac{\sqrt{3}}{2} + 2$

(c) $\dfrac{3\sqrt{3}}{2} + 1$

(d) $\dfrac{5}{2} + \dfrac{1}{3}$

(e) 1

18. The area of the region bounded by the curves $x = -y^2$, $y = x - 4$, $y = -1$ and $y = 2$ is

(a) $33/2$

(b) $35/2$

(c) $37/2$

(d) $29/2$

(e) $39/2$

19. The base of a certain solid is the region enclosed by $y = \sqrt{x}$, $y = 0, x = 0$, and $x = 1$. If the cross sections perpendicular to the x-axis are semicircles, then the volume of this solid is

(a) $\pi/16$

(b) $\pi/4$

(c) $\pi/8$

(d) $\pi/2$

(e) $\pi/10$

20. The volume of the solid generated by rotating the region enclosed by $y = \sin x, y = 0$, $x = 0$, and $x = \frac{\pi}{2}$ about the line $x = -1$, is given by the integral

(a) $\int_0^1 \pi \left(\left(1 + \frac{\pi}{2}\right)^2 - \left(1 + \sin^{-1} y\right)^2 \right) \, dy$

(b) $\int_0^1 \pi \left(\sin^{-1} y - \frac{\pi}{2} \right) \, dy$

(c) $\int_0^1 \pi \left(\sin^{-1} x - \frac{\pi}{2} \right) \, dx$

(d) $\int_0^2 \pi \left(\left(1 + \sin^{-1} y\right)^2 - \left(\frac{\pi}{2}\right)^2 \right) \, dy$

(e) $\int_0^1 \pi \left(\left(1 + \frac{\pi}{2}\right)^2 - \left(\sin^{-1} y\right)^2 \right) \, dy$