King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
**Math 101(12) Class Test II (B) Summer 2018(173)**

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(1) Assume that

$$f(x) = \begin{cases} 5 - \frac{1}{6}x & x < -2 \\ \frac{6}{x-4} & x \geq -2. \end{cases}$$

Determine if $f$ is differentiable at $x = -2$; i.e. determine $f'(-2)$ if it exists.

(2) If $f$ is differentiable at $a$, where $a > 0$, evaluate the following limit.

$$\lim_{x \to a} \frac{\cos x - \cos a}{\sqrt{x} - \sqrt{a}}.$$

(3) Find $y'$ of each of the following:

(a) $y = (\csc x)^{-1} \csc^{-1} x$.

(b) $y = \frac{1+\tan^{-1} x}{2-3\tan^{-1} x}$.

(c) $y = \pi^2 + 2^x + x^2 + x^{1/x} + \frac{\sqrt{x^{101}-101}}{2x^3-e^x}$

(4) Assume that $h(x) = (f(x^{-1}))^3$, where $f$ is a differentiable function. If $f(\frac{1}{2}) = 1$ and $f'(\frac{1}{2}) = -1$ , determine an equation of the line tangent to the graph of $h$ at $x = 2$.

(5) A stone dropped in a pond sends out a circular ripple whose radius increases at a constant rate of 4 ft/sec. After 12 seconds, how rapidly is the area inclosed by the ripple increasing?

(6) A race car travels almost $\frac{1}{2}$ km in 10 seconds, its distance from the start in meter after $t$ seconds being $f(t) = t^3 - 6t^2 + 9t$. How fast was it going halfway down the track? (Hint: $f(t) = 249$ when $t \approx 8.43$)

(7) Find (i) $\lim_{x \to 0} \frac{\sin(\sin x)}{x}$, (ii) $\lim_{n \to \infty} (2n) \ln(1 + \frac{1}{5n})$.

(8) Find the formula for the n$^{th}$ derivative $y^{(n)}$ of $y = 1/(x+1)$.

(9) Evaluate $\frac{dz}{dx}|_{x=0}$ if $z = \sin(e^{\sqrt{u}})$ and $u = 2\sin(x + \pi/6)$.