Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
STAT460: Time Series
Dr. Mohammad H. Omar
Final Exam Term 172 FORM A SOLUTION
Saturday May 5 2018
7.00pm-10.00pm

Name:_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 180 minutes, GOOD LUCK, and you may begin now!

Part	Question	Total Marks	Marks Obtained	Comments
A	1	4+2+3+5=14		
	2	4		
	3	4+4+2 = 10		
	4	3+3+3 = 9		
	5	5+5 = 10		
	6	3+3+3 = 9		
	7	4+5 = 9		
B	8	4+5 = 9		
	9	5+5+2 = 12		
	10	8+2+2+4=16		
	Total	100	100	

Extra blank page

Part A. Concepts

1. (4+2+3+5=14 points) Let $\{Y_t\}$ be quarterly data from a special ARIMA process of the form $Y_t = \phi Y_{t-4} + e_t$.

a) Find the **range of values** of ϕ for which the process is stationary.

b) Write the **correct order** (p and q) of this stationary ARIMA process.

c) Write an **alternative** but identical seasonal **model** to your answer to (b) above.

d) Express this model in the **general linear process** form to find ψ_4 and ψ_5.

2. (4 points) A stationary time series of length 169 produced sample partial autocorrelations of $\hat{\phi}_{11} = 0.9$, $\hat{\phi}_{22} = -0.7$, $\hat{\phi}_{33} = 0.09$, and $\hat{\phi}_{44} = 0.00$. On the basis of this information alone, what model should be tentatively specified for this series?

3. (4+4+2=10 points) The following partial data from tempdub file provides the monthly temperature for 24 months from Jan 1973 to December 1975 in Dubuque, Iowa.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	22.5	25.7	42.3	45.2	55.5	68.9	72.3	72.3	62.5	55.6	38.0	20.4
1974	17.6	20.5	34.2	49.2	54.8	63.8	74.0	67.1	57.7	50.8	36.8	25.5
1975	20.4	19.6	24.6	41.3	61.8	68.5	72.0	71.1	57.3	52.5	40.6	26.2

With these data, the following outputs were obtained:

Output A

Coefficient	Estimate	Std. Error	t-value	Pr(>|t|)
January	20.167	2.177	9.265	2.14e-09 ***
February	21.933	2.177	10.076	4.24e-10 ***
March	33.700	2.177	15.482	5.44e-14 ***
April	45.233	2.177	20.780	< 2e-16 ***
May	57.367	2.177	26.354	< 2e-16 ***
June	67.067	2.177	30.810	< 2e-16 ***
July	72.767	2.177	33.429	< 2e-16 ***
August	70.167	2.177	32.235	< 2e-16 ***
September	59.167	2.177	27.181	< 2e-16 ***
October	52.967	2.177	24.333	< 2e-16 ***
November	38.467	2.177	17.672	2.91e-15 ***
December	24.033	2.177	11.041	6.87e-11 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
Residual standard error: 3.77 on 24 degrees of freedom
Multiple R-squared: 0.9963, Adjusted R-squared: 0.9944
F-statistic: 535.4 on 12 and 24 DF, p-value: < 2.2e-16

Output B

Coefficient	Estimate	Std. Error	t-value	Pr(>|t|)
Intercept	20.167	2.177	9.265	2.14e-09 ***
February	1.767	3.078	0.574	0.571380
March	13.533	3.078	4.396	0.000193 ***
April	25.067	3.078	8.143	2.30e-08 ***
May	37.200	3.078	12.084	1.08e-11 ***
June	46.900	3.078	15.235	7.73e-14 ***
July	52.600	3.078	17.087	6.17e-15 ***
August	50.000	3.078	16.242	1.90e-14 ***
September	39.000	3.078	12.669	4.03e-12 ***
October	32.800	3.078	10.655	1.40e-10 ***
November	18.300	3.078	5.945	3.90e-06 ***
December	3.867	3.078	1.256	0.221182

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
Residual standard error: 3.77 on 24 degrees of freedom
Multiple R-squared: 0.9725, Adjusted R^2: 0.9599
F-statistic: 77.19 on 11 and 24 DF, p-value: 5.667e-16

a) Write the model estimated by both outputs.
Output A Model: _____.

Output B Model: _____.

b) Compare the results for the month of December in output A to output B. Explain any discrepancy in results.

c) Explain why the models in output A and B above are less preferred compared to a seasonal ARIMA model.

_____
_____
_____
_____
_____

4. (3+3+3 = 9 points) Assuming that the following data arise from a stationary process, calculate method of moments estimates of μ, γ_0, and ρ_1:

6	5	4	6	4

5. (5+5 = 10 points) Based on a series of length n = 200, we fit an AR(2) model and obtain residual correlations of $\hat{r}_1 = 0.13,\ \hat{r}_2 = 0.13,\ and\ \hat{r}_3 = 0.12$. If $\hat{\phi}_1 = 1.1$ and $\hat{\phi}_2 = -0.8$, do these residual autocorrelations
   a) ***individually*** provide support for AR(2) specification?
   b) ***jointly*** provide support for AR(2) specification? (Hint: Use the Ljung-Box test)

6. (3+3+3 = 9 points) For an AR(1) model with $Y_t = 12.2$, $\phi = -0.5$, and $\mu = 10.8$.
   a) Find $\hat{Y}_t(1)$.
   b) Calculate $\hat{Y}_t(2)$.
   c) Find $\hat{Y}_t(10)$.

7. (4+5 = 9 points) An AR model has AR characteristic polynomial
$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$$
   a) Is the model stationary?
   b) Identify the model as a certain seasonal ARIMA model and write the model.

Name:_____ ID#:_____ Serial #:___

Part B. Analysis.

Direction: Use *R* software to conduct appropriate analysis to answer the following questions. Be sure to save your important outputs and graph into MSWORD file under your name and email this file to the instructor at the end of the exam.

8. (4+5=9 points) The data file named "deere3" contains 57 consecutive measurements recorded from a complex machine tool at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced.
   a) Display the time series plot of this series and comment on its appearance. Would a stationary model be appropriate here?
   b) Display the sample ACF and PACF for this series to select tentative orders for an ARMA model for the series.

9. (5+5+2 = 12 points) The data file named "robot" contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series.
   a) Estimate the parameters of an AR(1) model for these data and complete the blanks below.

	ar1	Intercept
		0.0015
s.e.	0.0528	

sigma^2 estimated as [ ]: log likelihood = [ ], aic = [ ].

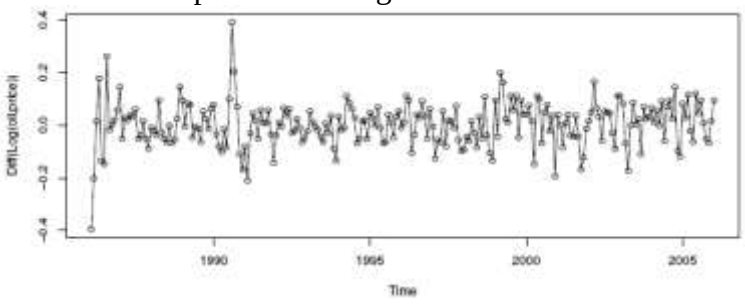
   b) Estimate the parameters of an IMA(1,1) model for these data

```
Coefficients:
        ma1
      [   ]
s.e.  [   ]

sigma^2 estimated as [ ]: log likelihood = [ ], aic = [ ].
```

   c) Comparing the results from parts (a) and (b) in terms of AIC, **decide** the **better model** for this data.

10. (8+2+2+4=16 points) The data file oil.price contains a time series of oil price. The time series plot of the difference of logarithms of the oil price series is given below for reference.



The difference of the logarithms looks fairly stable except for possible outliers at the beginning (February 1986) and at August 1990.

a) The EACF plot for the difference of logarithms of oil price series suggests an AR(1) or possibly an AR(4) model.

1) Estimate both these models using maximum likelihood and complete the output below.

```
Coefficients:
         ar1
      [      ]
s.e.  0.0660

sigma^2 estimated as [          ]: log likelihood = 258.55, aic = -515.11
```

```
Coefficients:
         ar1     ar2     ar3     ar4
      0.2673 -0.1550 [      ] [      ]
s.e.  0.0669  0.0691  0.0691   0.0681

sigma^2 estimated as 0.006603: log likelihood = 261.82, aic = [      ]
```

2) Do an overfit analysis of the better of the two models and ***appropriately*** complete the output below.

```
Coefficients:
          ar1      ar2      ar3      ar4      ar5
      [      ] [      ] [      ] [      ] [      ]
s.e.  [      ] [      ] [      ] [      ] [      ]

sigma^2 estimated as [      ]: log likelihood = [      ], aic = [      ]
```

b) The ACF of the difference of logarithms of oil price series suggests an MA(1) model. Estimate this model by maximum likelihood and complete the output below.

```
Coefficients:
          ma1
      [      ]
s.e.  [      ]

sigma^2 estimated as 0.006689: log likelihood = [      ], aic = [      ]
```

c) Which of the models, (AR(1), AR(4), the overfit, or MA(1)) would you prefer given results in (a) and (b) above?

d) Perform diagnostic tests for the preferred model in (c).