Revised Simplex Method

EXAMPLE 2.

$$\max \; 3x_1 + x_2 + 2x_3$$

s.t.

$$\begin{cases} x_1 + x_2 + 3x_3 \leq 5 \\ 2x_1 + x_2 + x_3 \leq 4 \end{cases}$$

$$x_1 ; x_2 ; x_3 \geq 0$$

Tab1.

	B^{-1}		a_1	y_1	$B^{-1}b$	y_{01}/y_1
e_1	1	0	1	1	5	5
e_2	0	1	2	2	4	2
r_N^t	3 1 2					

$$\lambda^t = c_B^t B^{-1} = (0\;0)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0 \;\; 0)$$

$$\Rightarrow r_N^t = (3 \;\; 1 \;\; 2)$$

$\Rightarrow x_1$ Enters $\Rightarrow e_2$ leaves.

Tab2.

	B^{-1}		$B^{-1}b$	a_3	y_3	y_{03}/y_3
e_1	1	$-1/2$	3	3	$5/2$	$6/5$
x_1	0	$1/2$	2	1	$1/2$	4
r_N^t	$-1/2$ $1/2$ $-3/2$					

$$\lambda^t = (0 \;\; 3)\begin{pmatrix} 1 & -1/2 \\ 0 & 1/2 \end{pmatrix} = (0 \;\; 3/2)$$

$$r_N^t = c_N^t - \lambda^t N = (1 \;\; 2 \;\; 0) - (0 \;\; \tfrac{3}{2})\begin{pmatrix} 1 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= (1 \;\; 2 \;\; 0) - (\tfrac{3}{2} \;\; \tfrac{3}{2} \;\; \tfrac{3}{2})$$

$$= (-\tfrac{1}{2} \;\; +\tfrac{1}{2} \;\; -\tfrac{3}{2})$$

$\Rightarrow x_3$ Enters

Tab3.

	B^{-1}		$B^{-1}b$
x_3	$2/5$	$-1/5$	$6/5$
x_1	$-1/5$	$3/5$	$7/5$
r_N^t	$-3/5$ $-1/5$ $-7/5$		

$$\lambda^t = (2 \;\; 3)\begin{pmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{pmatrix}$$

$$\lambda^t = (1/5 \;\; 7/5)$$

$$r_N^t = (1 \;\; 0 \;\; 0) - (1/5 \;\; 7/5)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$= (1 \;\; 0 \;\; 0) - (8/5 \;\; 1/5 \;\; 7/5)$$

$$= (-3/5 \;\; -1/5 \;\; -7/5)$$

$$B^{-1} \cdot b = \begin{pmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 6/5 \\ 7/5 \end{pmatrix}$$

optimal solution is

$$x_1 = \tfrac{7}{5} ; x_3 = \tfrac{6}{5}$$

$$z^* = \tfrac{33}{5}$$

EXAMPLE 3

$$\min\ 2x+3y+z$$

s.t.
$$\begin{cases} x-y+z\geq 3\\ x+y-z\geq 2\\ -x+y+z\geq 3\end{cases} \Rightarrow \begin{cases} x-y+z-e_1=3\\ x+y-z-e_2=2\\ -x+y+z-e_3=3\end{cases}$$

$x,y,z\geq 0$ $\qquad$ $x,y,z\geq 0$

$e_1; e_2; e_3 \geq 0$

	B^{-1}			a_3	y_3	$B^{-1}b$	
a_1	1	0	0	1	1	3	3
a_2	0	1	0	-1	-1	2	—
a_3	0	0	1	1	1	3	3

$$\lambda^t=(M\ M\ M)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=(M\ M\ M)$$

$$r_N^t=(2\ 3\ 1\ 0\ 0\ 0)-(M\ M\ M)\begin{pmatrix}1&-1&1&1&0&0\\1&1&-1&0&1&0\\-1&1&1&0&0&1\end{pmatrix}$$

$$=(2-M\ \ 3-M\ \ 1-M\ \ldots)$$

z enters.

	B^{-1}			a_2	y_2	$B^{-1}b$
z	1	0	0	-1	-1	3
a_2	1	1	0	1	0	5
a_3	-1	0	1	1	2	0

$$\lambda^t=(1\ M\ M)\begin{pmatrix}1&0&0\\1&1&0\\-1&0&1\end{pmatrix}=(1\ M\ M)$$

$$r_N^t=(2\ 3\ 0\ \cdots\ 0)-(1\ M\ M)\begin{pmatrix}1&-1&\cdots\\1&1&0\\-1&1&0\end{pmatrix}$$

$$=(2\ 3\ 0\ \cdots\ 0)-(1\ \ 2M-1\ \ 0\ \cdots\ 0)$$

$$=(1\ \ 4-2M\ \cdots)$$

y enters.

	B^{-1}			a_1	y_1	$B^{-1}b$
z	1/2	0	1/2	1	0	3
a_2	1	1	0	1	2	5
y	-1/2	0	1/2	-1	-1	0

$$\lambda^t=(1\ M\ 3)\begin{pmatrix}1/2&0&1/2\\1&1&0\\-1/2&0&1/2\end{pmatrix}=(M-1\ \ M\ \ 2)$$

$$r_N^t=(2\ 0\ \cdots\ 0)-(M-1\ \ M\ \ 2)\begin{pmatrix}1&-1&\\1&&-1\\-1&&&-1\end{pmatrix}$$

$$=(2\ 0\ \cdots\ 0)-(2M-3$$

x enters

	B^{-1}			$B^{-1}b$
z	1/2	0	1/2	3
x	1/2	1/2	0	5/2
y	0	1/2	1/2	5/2

$$\lambda^t=(2\ 3\ 1)\begin{pmatrix}1/2&0&1/2\\1/2&1/2&0\\0&1/2&1/2\end{pmatrix}=\left(\tfrac{5}{2}\ \ 2\ \ \tfrac{3}{2}\right)$$

$$r_N^t=(0\ 0\ 0\ M\ M\ M)-\left(\tfrac{5}{2}\ 2\ \tfrac{3}{2}\right)\begin{pmatrix}-1&0&0&1&0&0\\0&-1&0&0&1&0\\0&0&-1&0&0&1\end{pmatrix}$$
$$=\left(\tfrac{5}{2}\ \ 2\ \ \tfrac{3}{2}\ \ M-\tfrac{5}{2}\ \ M-2\ \ M-\tfrac{3}{2}\right)$$

The optimal sol^{o} is

$x=5/2\quad y=5/2\quad z=3$

$$Z^*=\frac{21}{2}$$