16':

Revised Simplex Method ⇒ Save computational effort.

Initialization:

Given the inverse B^{-1} of a current basis → $x_B = B^{-1}.b = y_0$.

Step 1.

- Calculate $r_N^t = C_N^t - \underbrace{C_B^t B^{-1}}_{\lambda^t} N$
- if $r_N^t \geq 0$ (≤ 0) stop.

Step 2.

- Determine which vector a_q would enter the basis
- Compute $y_q = B^{-1} a_q$ → obtain a_q in terms of the current basis.

Step 3

- If no $y_{iq} > 0$ → problem unbounded.
- Else $\min \left\{ \frac{y_{i0}}{y_{iq}} \right\}$ for $y_{iq} > 0$ → determine which vector would leave the basis.

Step 4.

Update B^{-1} and $B^{-1}b$. Return to Step 1.

Revised Simplex Method.

Example 1:

$$\max\ 3x_1 + x_2 + 3x_3$$

s.t.

$$\begin{cases} 2x_1 + x_2 + x_3 \le 2 \\ x_1 + 2x_2 + 3x_3 \le 5 \\ 2x_1 + 2x_2 + x_3 \le 6 \end{cases}$$

All $x_i \geq 0$.

Tab1.

B^{-1}			a_1	y_1	b	ratio
1	0	0	2	2	2	1
0	1	0	1	1	5	5
0	0	1	2	2	6	3
r_N: 3	1	3				

Tab2

	B^{-1}			a_2	y_2	b	$B^{-1}b$	
x_1	1/2	0	0	1	1/2	2	1	2
e_2	-1/2	1	0	2	3/2	5	4	8/3
e_3	-1	0	1	2	1	6	4	4
r_N	-1/2	3/2	-3/2					

reduced costs.

$$\lambda^t = c_B^t B^{-1}$$

$$\lambda^t = \underbrace{[3\ 0\ 0]}_{c_B^t} \begin{bmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & 0 & 0 \end{bmatrix}$$

$$\rightarrow r_N^t = c_N^t - \lambda^t N = (1\ 3\ 0) - (3/2\ 0\ 0)\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= (1\ 3\ 0) - (3/2\ 3/2\ 3/2)$$

$$r_N^t = (-1/2\ \ 3/2\ \ -3/2)$$

tab3:

	B^{-1}			a_3	y_3	b	$B^{-1}b$
x_2	1	0	0	1	1	2	2
e_2	-2	1	0	3	1	5	1
e_3	-2	0	1	1	-1	6	2
r_N^t	1	2	-1				

reduced costs.

$$\lambda^t = \underbrace{[1\ 0\ 0]}_{c_B^t} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}}_{B^{-1}} = [1\ 0\ 0]$$

$$\rightarrow r_N^t = c_N^t - \lambda^t N$$

$$= (3\ 3\ 0) - (1\ 0\ 0)\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= (3\ 3\ 0) - (2\ 1\ 1) = (1\ 2\ -1).$$

tab4:

	B^{-1}			a_1	y_1	b	$B^{-1}b$
x_2	3	-1	0	2	5	2	1
x_3	-2	1	0	1	-3	5	1
e_3	-4	1	1	2	-5	6	3
r_N^t	7	3	-2				

$$\lambda^t = \underbrace{[1\ 3\ 0]}_{c_B^t} \underbrace{\begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -4 & 1 & 1 \end{bmatrix}}_{B^{-1}} = [-3\ 2\ 0]$$

$$\rightarrow r_N^t = c_N^t - \lambda^t N$$

$$= (3\ 0\ 0) - (-3\ 2\ 0)\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}$$

$$= (3\ 0\ 0) - (-4\ -3\ 2)$$

$$= (7\ 3\ -2)$$

tab5

		B^{-1}			a	b	$B^{-1}b$
3	x_1	3/5	-1/5	0		2	1/5
3	x_3	-1/5	2/5	0		5	8/5
	e_3	-1	0	1		6	4

$$Z^* = 27/5$$

$$\lambda^t = \underbrace{[3\ 3\ 0]}_{c_B^t} \underbrace{\begin{pmatrix} 3/5 & -1/5 & 0 \\ -1/5 & 2/5 & 0 \\ -1 & 0 & 1 \end{pmatrix}}_{B^{-1}} = (6/5\ \ 3/5\ \ 0)$$

$$r_N^t = c_N^t - \lambda^t N$$

$$= (1\ 0\ 0) - (6/5\ 3/5\ 0)\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix}$$

$$= (1\ 0\ 0) - (12/5\ \ 6/5\ \ 3/5)$$

$$= (-7/5\ \ -6/5\ \ -3/5) \quad \text{All} \le 0$$

$\rightarrow$ optimal Solution

$$x_1^* = 1/5,\ x_3^* = \frac{8}{5};\ Z^* = \frac{27}{5}$$