**King Fahd University of Petroleum and Minerals**
**Department of Mathematics and Statistics**
**STAT-361 Operations Research I** [1]
**Final Exam**
Four Problems, May $14^{th}$, 2018 [2]

**Problem** 1 **(**25 **pts)**

Consider the following linear program (P):

$$\begin{array}{ll} \min\limits_{x_1,x_2} & 3x_1 + 3x_2 \\ \text{s.t.} & 4x_1 + 3x_2 \geq 4, \\ & 3x_1 + 4x_2 \geq 4, \\ & x_1, x_2 \geq 0. \end{array}$$

(a) **Solve** the linear program (P) graphically. (5 points)

[1] Dr. Slim Belhaiza (c)
[2] This is an open book exam. The exam lasts 120 minutes.

(b) **Formulate** the dual linear program (D) associated to the program (P) and **solve** it using the Revised Simplex Method. (15 points)

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(c) Write few words to explain that the solutions in (a) and (b) match. (5 points)

**Problem** 2 **(**25 **pts)**

Consider the following linear program:

$$\begin{array}{ll} \max\limits_{x_1,x_2,x_3} & 5x_1 + 3x_2 + 4x_3 \\ \text{s.t.} & x_1 + 2x_2 + 2x_3 \leq 3, \\ & x_1 + 2x_2 + x_3 \geq 4, \\ & 3x_1 + x_2 + x_3 \geq 5, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

Solve the linear program using the Dual Simplex algorithm.

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**Problem** 3 **(25 Points)**

Consider the following transportation problem. The problem involves 3 plants supplying 4 customer zones. The following table 1 displays the unit transportation costs, the supplies and the demands. Find the optimal solution to this problem.

| Demand Nodes → | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| Supply Nodes ↓ | | | Costs | | Offer |
| 1 | 3 | 3 | 3 | 6 | 700 |
| 2 | 1 | 4 | 2 | 5 | 800 |
| 3 | 5 | 2 | 5 | 4 | 600 |
| Demand | 500 | 500 | 500 | 600 | |

Table 1: Data for problem 3

You can use the following tables. You have to use the **Least Cost** initialization method, otherwise you are subject to $-5$ points.

**Interation** 1

| Demand Nodes | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | | | | | 700 |
| 2 | | | | | 800 |
| 3 | | | | | 600 |
| Demand | 500 | 500 | 500 | 600 | |

**Interation** 2

| Demand Nodes | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | | | | | 700 |
| 2 | | | | | 800 |
| 3 | | | | | 600 |
| Demand | 500 | 500 | 500 | 600 | |

**Interation** 3

| Demand Nodes | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | | | | | 700 |
| 2 | | | | | 800 |
| 3 | | | | | 600 |
| Demand | 500 | 500 | 500 | 600 | |

**Interation** 4

| Demand Nodes | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | | | | | 700 |
| 2 | | | | | 800 |
| 3 | | | | | 600 |
| Demand | 500 | 500 | 500 | 600 | |

**Interation** 5

| Demand Nodes | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | | | | | 700 |
| 2 | | | | | 800 |
| 3 | | | | | 600 |
| Demand | 500 | 500 | 500 | 600 | |

**Problem** 4 **(**25 **pts)**

Consider the following project scheduling problem detailed in table 2.

| Tasks | Condition | Duration (days) |
|---|---|---|
| a | – | 4 |
| b | – | 6 |
| c | after a | 3 |
| d | after b | 4 |
| e | after c and d | 5 |
| f | after e | 3 |
| g | after d | 4 |
| h | after f and g | 6 |
| i | after g and h | 5 |
| k | after h and i | 4 |

Table 2: Data for problem 4

(*i*).Draw the graph representing the interdependence between the tasks of the project. (10 pts)
(*ii*).Find the shortest possible duration of the project. (10 pts)
(*iii*).Find the critical tasks and the critical tasks. (5 pts)