**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**DEPARTMENT OF MATHEMATICS & STATISTICS**
**Term 172**
**STAT 319 Exam #1**

Name: ______________________________ ID #: ______________

- State all assumptions and conditions needed, otherwise you lose marks.
- Show all details.
- If there is a rule or a formula you are using, write down that rule or formula.
- Answers without justification are not accepted.

Some Useful Formulas

- Confidence Intervals

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \quad or \quad \bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \quad or \quad \bar{X} \pm t_{\frac{\alpha}{2},n-1} \frac{S}{\sqrt{n}} \quad or \quad \hat{p} \pm z_{\frac{\alpha}{2},} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Test Statistics $\frac{(\bar{X}-\mu)}{\sigma/\sqrt{n}} \quad or \quad \frac{(\bar{X}-\mu)}{S/\sqrt{n}} \quad or \quad \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

- Sample Sizes $\left(\frac{Z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 \quad or \quad \left(\frac{Z_{\frac{\alpha}{2}}\, s}{E}\right)^2 \quad or \quad \frac{1}{4}\left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2 \quad or \quad \left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2 \hat{p}(1-\hat{p})$

| Question | Maximum Marks | Marks Obtained |
|---|---|---|
| 1 | 8 | |
| 2 | 3 | |
| 3 | 3 | |
| 4 | 8 | |
| 5 | 3 | |
| Total | 25 | |

1) A factory has a large-scale production of axle shafts. In a random sample of 500 shafts 25 are found defective.

   a) Construct a 95% confidence interval for the proportion of non-defective shafts.

   *(3 marks)*

   b) Use the confidence interval above to test the hypothesis that the proportion of non-defective shafts differs from 0.94. State your hypotheses, decision and conclusion.

   *(3 marks)*

   c) At what significance level are you testing the hypothesis in b)? *(1 mark)*

   d) The management decides that error in estimation must not exceed 0.01, with 95% confidence. Assuming the estimate obtained in a) above, what sample size should be used to estimate the proportion of non-defective items? *(1 mark)*

2) It is believed that 20% of KFUPM prep-year students choose mechanical engineering as their major. A random sample of 80 such students is chosen. Approximate the probability that more than 25 students prefer mechanical engineering. *(3 marks)*

3) Assume that the time to failure, in hours, of a newly developed electronic device follows a lognormal distribution with parameters $\theta = -1$ and $\omega^2 = 22$. The density of the lognormal is

$$f(x) = \begin{cases} \dfrac{1}{x\omega\sqrt{2\pi}} e^{-\frac{1}{2\omega^2}[\ln(x)-\theta]^2} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

Find the probability that the device lasts longer than 100 hours without failure.

*(3 marks)*

4) An electronic parts factory produces resistors. Assume the resistance follows a distribution with standard deviation 0.156 ohms. A random sample of 60 resistors has an average resistance of 0.45 ohms.

a) Is there significant evidence that the population mean resistance is less than 0.5 ohms? Use $\alpha = 0.02$ *(4 marks)*

b) Find the p-value of the test in a). *(1 mark)*

c) Express the rejection region of the above test in terms of the sample mean. *(1 mark)*

d) Find the probability of type II error if the actual resistance is 0.48. *(2 marks)*

5) A study of a chemical factory's discharge of pollutants over 25 randomly selected days shows that the mean daily discharge was 25 tons with standard deviation 8.5 tons.

   a) Construct a 99% confidence interval for the true mean of the factory's discharge of pollutants. *(2 marks)*

   b) What assumptions, if any, are needed to construct the confidence interval above? *(1 mark)*