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KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
Term 172
STAT 319 Exam #1

Name: Key ID #: ____________

- State all assumptions needed, otherwise you lose marks
- Show all details.
- If there is a rule you are using, write down that rule.
- Answers without justification are not accepted.

Some Useful Formulas

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0, P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P(B|A_j)}{\sum_{i=1}^{k} P(A_i)P(B|A_i)}, \quad j = 1, 2, \ldots, k$
- $\mu = E(X) = \sum xP(X = x) \quad or \quad \mu = E(X) = \int xf(x)dx,$
- $\sigma^2 = E(x-\mu)^2 = E(x)^2 - (E(X))^2$
- $P(X = x) = C_x^n p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n, \quad \mu = np \;\&\; \sigma = \sqrt{np(1-p)}$
- $P(X = x) = \frac{C_x^K C_{n-x}^{N-K}}{C_n^N}, \quad x = \{0, \cdots, min(K, n)\}, \quad \mu = n\frac{K}{N} \;\&\; \sigma = \sqrt{n\frac{K}{N}\left(1 - \frac{K}{N}\right)} \cdot \sqrt{\frac{N-n}{N-1}}$
- $P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, x = 0, 1, 2, \ldots, \infty; \quad \mu = \lambda t \;\&\; \sigma = \sqrt{\lambda t}$
- $f(x) = \frac{1}{b-a}, \quad a \le x \le b; \quad \mu = \frac{b+a}{2} \;\&\; \sigma = \sqrt{\frac{(b-a)^2}{12}}$
- $f(x) = \lambda e^{-\lambda x}, x > 0; \; F(x) = 1 - e^{-\lambda x}, \; \mu = \frac{1}{\lambda} \;\&\; \sigma = \frac{1}{\lambda}$
- $f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}}, \; x > 0; \; F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}$

  $\mu = \delta\Gamma\left(1 + \frac{1}{\beta}\right) \;\&\; \sigma^2 = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \mu^2$

Question	Maximum Marks	Marks Obtained
1	3	
2	5	
3	3	
4	8	
5	6	
Total	25	

1) In a reliability test there is a 42% probability that a computer chip survives more than 500 temperature cycles. If a computer chip does not survive more than 500 temperature cycles, then there is a 73% probability that it was manufactured by company A. What is the probability that a computer chip is not manufactured by company A and does not survive more than 500 temperature cycles? (3 marks)

Let B be the event that the chip survives ; $P(B) = 0.42$

Let A be the event that the chip is manufactured by company A

$P(A|B') = 0.73$

$P(A' \cap B') = P(A'|B') \cdot P(B')$ ← 1 mark

$= (1-0.73)(0.58)$ ← 1 mark

$= (0.27)(0.58)$

$= 0.1566$ ← 1 mark.

If no details then no marks

2) An industrial psychologist has developed a test to identify people with potential to be good managers. His statistics indicate that 30 percent of the employees in the company have the potential to be good managers. When his test is given to a large number of employees similar to the ones in his company, 80 percent of the potentially good managers will pass the test while only 20 percent of the potentially poor managers will pass it. What is the probability that an employee who passes the test will be a good manager? (5 marks)

mark
Let G = event that employee has potential to be a good manager; $P(G) = 0.30$

Let P = event that employee passes the test; $P(P|G) = 0.80$

$P(P|G') = 0.20$

$P(G|P) = \dfrac{P(P \cap G)}{P(G)}$ ← 1 mark

$= \dfrac{P(P|G) \cdot P(G)}{P(P|G) \cdot P(G) + P(P|G') \cdot P(G')}$ Bayes Rule ← 1 mark

1 mark → $= \dfrac{(0.8)(0.3)}{(0.8)(0.3) + (0.2)(0.7)} = \dfrac{0.24}{0.38} = 0.63$ ← 1 mark

3) A rental car facility has 10 foreign cars and 15 domestic cars waiting to be serviced on a particular Saturday morning. Because there are so few mechanics working on Saturday, only 6 cars can be serviced. If the 6 cars are chosen at random, what is the probability that at least 2 of the selected cars are domestic? (3 marks)

N = Total number of cars = $15 + 10 = 25$

K = number of domestic cars = 15.

n = number of selected cars = 6

1 mark

Let X = # selected domestic cars

mark → $P(X \geq 2) = 1 - P(X < 2)$

1 mark → $= 1 - [P(X=0) + P(X=1)] = 1 - \left[\dfrac{\binom{10}{6} \cdot \binom{15}{0}}{\binom{25}{6}} + \dfrac{\binom{10}{5}\binom{15}{1}}{\binom{25}{6}} \right]$

$= 1 - (0.0012 + 0.0213) = 0.9775$.

4) The total amount of gas that is pumped at a station in a month can be modelled with a random variable X (measured in 10000 gallons) with probability density function

$$f(x) = \begin{cases} x/2 & 0 \le x \le 1 \\ (4-x)/6 & 1 < x \le 4 \\ 0 & elsewhere \end{cases}$$

a) Find the probability that the station will pump between 9000 and 11000 gallons in a particular month. (3 marks)

$$P(0.9 \le X \le 1.1) = \int_{0.9}^{1} \frac{x}{2}\,dx + \int_{1}^{1.1} \frac{4-x}{6}\,dx \quad \leftarrow \text{1 mark}$$

$$= 0.0475 + 0.0492 \quad \leftarrow \text{1 mark}$$

$$= 0.0967 \quad \leftarrow \text{1 mark}$$

b) If in a month, the station pumped more than 10000 gallons, what is the probability that they pumped more than 15000 gallons during the same month? (3 marks)

$$P(X > 1.5 \mid X > 1) = \frac{P(X > 1.5)}{P(X > 1)} \quad \text{1 mark}$$

This can be solved graphically

$$= \frac{\int_{1.5}^{4} \frac{4-x}{6}\,dx}{1 - P(X \le 1)} \quad \text{1 mark}$$

$$= \frac{0.5208}{0.75} = 0.6944 \quad \text{1 mark}$$

c) Find the expected number of gallons of gas pumped per month. (2 marks)

$$E(x) = \int_{0}^{1} \frac{x^2}{2}\,dx + \int_{1}^{4} \frac{x(4-x)}{6}\,dx \quad \frac{1}{2}\text{ mark}$$

$$\left.\begin{aligned} &= \frac{1}{6} + \frac{3}{2} \\ &= 1.6667 \end{aligned}\right\} \text{1 mark}$$

Expected number of gallons pumped per month is 16667 $\frac{1}{2}$ mark

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5) A sugar refinery has three processing plants, all of which receive sugar in bulk. The amount of sugar that one plant can process in one day can be modeled as having an exponential distribution with mean of 4 tons, for each of the three plants.

a) Find the probability that a plant selected randomly will process more than 4 tons on a given day. (2 marks)

X = amount of sugar ; $f(x) = \frac{1}{4} e^{-x/4}$; $x > 0$ (1 mark)

$$P(X > 4) = \int_4^{\infty} \frac{1}{4} e^{-x/4} \, dx = e^{-1} = 0.3678 \quad (1 \text{ mark})$$

b) If a plant selected randomly, how much raw sugar should be stocked for that plant each day so that the chance of running out of product is only 0.05? (2 marks)

Let s be the amount that should be stocked ($\frac{1}{2}$ mark)

$P(X > s) = 0.05$ ($\frac{1}{2}$ mark)

$$e^{-s/4} = 0.05 \Rightarrow \underbrace{s = -4 \ln(0.05)}_{\frac{1}{2} \text{ mark}} = 11.98 \text{ tons.}$$

$\frac{1}{2}$ mark

c) If the plants operate independently, find the probability that exactly two of the three plants will process more than 4 tons on a given day? (2 marks)

Let Y = # of plants that process more than 4 tons.

$Y \sim B(3, 0.3678)$

1 mark

$$P(Y = 2) = \binom{3}{2} (0.3678)^2 (0.6322) \quad \tfrac{1}{2} \text{ mark}$$

$$= 0.2566 \quad \tfrac{1}{2} \text{ mark}$$