KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 301: Introduction to Probability Theory
Semester 172, Third Major Exam
Thursday April 26, 2018 (2:00 pm)

Name: ID #:

Question No	Full Marks	Marks Obtained
1	**12**	
2	**44**	
3	**07**	
4	**10**	
5	**07**	
Bonus	**08**	
Total	**80**	

Instructions:

1. Mobiles are not allowed in exam. If you have your **mobile** with you, **turn it off** and put it **under your seat** so that it is visible to proctor.
2. Make sure you have 14 unique pages of exam paper (including this title page).
3. Show all the calculation steps. There are points for the steps so if your miss them, you would lose points.

Q.No.1: - (5+3+4 = 12 points) Suppose that there is a lot of 13 people out of which 8 are willing to buy the health insurance and the rest of 5 are not willing to do so. We randomly pick 2 people without replacement. Let X_1 equals 1 if the first customer picked is willing to buy the health insurance, otherwise 0. Similarly, X_2 equals 1 if the second customer picked is willing to buy the health insurance, otherwise 0.

(i) Give the joint probability mass function of X_1 and X_2.

(ii) Find $P(2X_1 + X_2 \leq 2)$.

(iii) Find the $E(X_1^2 X_2)$.

Q.No.2: - (2+6+6+8+3+4+8+7 = 44)

Let X and Y are jointly continuous with a joint pdf $f(x,y) = \begin{cases} 8xy & \text{if } 0 < x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(i) Verify that $f(x,y)$ is a valid joint *pdf*.

(ii) Find the marginal *pdf* of X and from there find $E(X)$ and $V(X)$.

(iii) Find the conditional *pdf* of X given $Y = y$ and from there find $E(X|Y = y)$ and $V(X|Y = y)$.

(iv) Find $E[E(X|Y=y)]$, $V[E(X|Y=y)]$ and $E[V(X|Y=y)]$.

(v) Verify that $E[E(X|Y=y)] = E(X)$.

(vi) Verify that $V(X) = V[E(X|Y=y)] + E[V(X|Y=y)]$

(vii) Find the *pdf* of $Z = X + Y$.

(viii) Find $\mathrm{Var}(X - Y)$.

Bonus Question: - (8 points) Find $P\left(\left|X-\frac{Y}{4}\right| \leq 0.1\right)$.

Note: Try this question only if you have time left after attempting all other questions.

Q.No.3: - (7 points) Let X and Y be independent exponential random variables with same λ. Find the joint moment generating function of $X + Y$ and $X - Y$.

Q.No.4: - (10 points) Ali and Hassan plan to meet at KFUPM mall to do STAT301 exam preparation together. Ali arrives at the mall at random time (uniform) between noon and 1:00 PM today; Hassan independently arrives at random time (uniform) between noon and 2:00 PM today. What is the probability that Ali has to wait for Hassan?

Q.No.5: - (7 points) If X and Y are independent Poisson random variables with respective paramters λ_1 and λ_2, calculate the conditional distribution of X given $X + Y = n$. Recognize the resulting distribution and its parameters.

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