
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 301: Introduction to Probability Theory
Semester 172, Second Major Exam
Wednesday March 27, 2018 (3:45 pm)

Name: _____

ID #: _____

Question No	Full Marks	Marks Obtained
1	14	
2	15	
3	15	
4	13	
5	11	
6	12	
Total	80	

Instructions:

1. Mobiles are not allowed in exam. If you have your **mobile** with you, **turn it off** and put it **under your seat** so that it is visible to proctor.
2. Make sure you have 14 unique pages of exam paper (including this title page).
3. Show all the calculation steps. There are points for the steps so if your miss them, you would lose points.

Q.No.1: - (3+2+4+5 = 14 points) A continuous random variable X has density function

$$f(x) = ke^{-\frac{|x|}{2}}; -\infty < x < \infty$$

(a) Find the value of k .

(b) Find $E(X)$.

(c) Find $P(|X| > 2)$

(d) Find $P(|X| > 2 \text{ given } |X| < 10)$

Q.No.2: - (5+5+5 = 15 points) Team A contains two players i.e. Ali and Hassan. It is known that Ali can distinguish correctly (just by tasting) between Pepsi and Coca-Cola 75% of the times. Hassan cannot tell the difference and, in fact, just guesses. Mr. Peter plays a game with Team A which is explained as follows:

Mr. Peter will throw a die and if a number less than 3 appears he chooses Ali, otherwise he chooses Hassan. Mr. Peter will give 5 glasses of drinks (each having filled with Pepsi or Coca-Cola, chosen by tossing a fair coin) to the chosen player. Team A wins the bet if their selected player gets 4 or more correct.

(a) What is the probability that Team A will win the game?

(b) If Team A wins a game, they get \$50 prize. Otherwise they lose \$30. Suppose that Mr. Peter plays this game with Team A two times. What is the expected amount that Team A will end up with?

(c) If Peter plays this game with Team A 5000 times, what is the probability that Team A will win at least 1700 games.

Q.No.3: - (6+4 +5= 15)

(a) If X is a geometric random variable, show mathematically that $P(X \leq a + b \mid X > a) = P(X \leq b)$ where a and b are two positive integers such that $a < b$.

(b) For a hypergeometric random variable Y , determine $\frac{P(Y=k+1)}{P(Y=k)}$. Simplify your result.

(c) For a negative binomial random variable W , determine $\text{var}(W)$.

Q.No.4: - (5+5+3 = 13 points)

(a) Let $X \sim \text{Beta}(a, b)$. Mathematically derive the mode of X .

(b) Let $Y \sim \text{Lognormal}(\theta, \omega^2)$. Mathematically derive the mean of Y .

(c) Let $W \sim \text{Uniform}(a, b)$. Mathematically derive the 70th percentile of W .

Q.No.5: - (3+4+4 = 11 points) The salaries of physicians in a certain specialty are approximately normally distributed. If 25 percent of these physicians earn less than \$180,000 and 30 percent earn more than \$320,000.

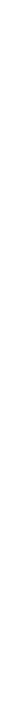
(a) What percent of the physicians earn less than \$200,000?



(b) What percent of the physicians earn between \$280,000 and \$330,000?



(c) What is the salary above which only 3 percent of the physicians earn?



Q.No.6: - (4+4+4 = 12 points) In a bad winter season (too cold), the probability of a person committing a suicide is .006 in a particular city; whereas in a good winter, this probability is .002 in that city. The probability that a winter season will be bad is 0.1. Assume that there are 500 people in that city in a winter season. Let X be the number of suicides and $B = \{\text{Bad winter}\}$.

Hint: Committing suicide is a rare event and the number of rare events are generally modeled by Poisson distribution.

(a) Calculate the probability of the event that there are no suicides.

(b) If there are no suicides, what is the probability that it was a bad winter season?

(c) What is the expected number of suicides in that winter season?

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Best of Luck