# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
# DEPARTMENT OF MATHEMATICS & STATISTICS
# DHAHRAN, SAUDI ARABIA

**STAT 301: Introduction to Probability Theory**
Semester 172, First Major Exam
Saturday February 24, 2018 (3:30 pm)

Name: ID #:

| Question No | Full Marks | Marks Obtained |
|---|---|---|
| 1 | **09** | |
| 2 | **16** | |
| 3 | **07** | |
| 4 | **12** | |
| 5 | **09** | |
| 6 | **12** | |
| **Total** | **65** | |

**Instructions:**

1. Mobiles are not allowed in exam. If you have your **mobile** with you, **turn it off** and put it **under your seat** so that it is visible to proctor.
2. Make sure you have 12 unique pages of exam paper (including this title page).
3. Show all the calculation steps. There are points for the steps so if your miss them, you would lose points.

Q.No.1: - (4+5 = 9 points) Mathematically prove that

(a) $$\binom{\boldsymbol{n}}{\boldsymbol{r}} = \binom{\boldsymbol{n-1}}{\boldsymbol{r-1}} + \binom{\boldsymbol{n-1}}{\boldsymbol{r}}$$

(b) $$k\binom{\boldsymbol{n}}{\boldsymbol{k}} = (n-k+1)\binom{\boldsymbol{n}}{\boldsymbol{k-1}} = n\binom{\boldsymbol{n-1}}{\boldsymbol{k-1}}$$

Q.No.2: - (10+6 = 16 points)

(a) A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be at least 3 complete pairs?

(b) Six balls are randomly drawn from an urn that contains 13 red, 14 blue and 15 green balls. Find the probability that either exactly 2 red balls or exactly 2 blue balls or exactly 2 green balls are withdrawn.

Q.No.3: - (3+4 = 7 points) 60% of the students at KFUPM take at least one of the two available Stat courses (Stat-I and Stat-II). 20% percent takes Stat-I but do not take Stat-II.

(a) If one of the students is chosen randomly, what is the probability that this student takes Stat-II?

(b) If a randomly picked student does not take Stat-II, what is the probability that he does not take Stat-I either?

Q.No.4: - (6+6 = 12 points)

(a) Mr. Hassan has asked his supervisor for a letter of recommendation for a new job. He estimates that there is a 70% chance that he will get the job if he receives a strong recommendation, a 30% chance that he will get the job if he receives a moderately good recommendation, and a 5% chance that he will get the job if he receives a weak recommendation. Hassan further estimates that the probabilities that the recommendations will be strong, moderate, and weak are 0.8, 0.18 and 0.02, respectively. Given that Mr. Hassan does not receive the job offer, how likely should he feel that he received a strong recommendation?

(b) Suppose that we pick one letter at random from the word BUBBLE and independently one letter at random from the word BURST.

(i) What is the probability that both letters are 'B'?

(ii) What is the probability that the two letters are the same?

Q.No.5: - (3+3+3 = 9 points)

(a) If 12 new teachers are to be divided among 3 schools such that each school must receive 4 teachers, how many divisions are possible?

(b) A student has to sell 3 books from a collection of 6 math, 7 science, 5 history and 4 economics books. How many choices are possible if the books are to be on different subjects?

(c) Four separate awards (best scholarship, best leadership qualities and so on) are to be presented to selected students from a class of 20. How many outcomes are possible if a student can receive any number of awards?

Q.No.6: - (5+7 = 12 points) Mathematically show that

(a) $P(A \mid B) = \{P(A \mid B \cap C) \times P(C \mid B)\} + \{P(A \mid B \cap \bar{C}) \times P(\bar{C} \mid B)\}$

(b) $P(A \cup B \cup C) = P(A \cup B) + P(\overline{A \cup B} \cap C)$