Math 568 - 172
HW # 4

Problem # 1 Solve the Cauchy problem

$$(P_1)\begin{cases} u_t(x,y,t) - 4\Delta u(x,y,t) - 3u = 8, & (x,t)\in\mathbb{R}^2\times(0,+\infty) \\ u(x,y,0) = xy, & (x,y)\in\mathbb{R}^2. \end{cases}$$

Hint: You may consider $v = u e^{\beta t}$, for an appropriate $\beta\in\mathbb{R}$.

Problem # 2 On the unit disk $D = \{(x,y) \,/\, x^2+y^2<1\}$, we consider the problem

$$(P_2)\begin{cases} u_t - \Delta u + u = 0, & \text{in } D_T = D\times(0,T). \\ u = 0, & \text{on } \partial D\times[0,T) \\ u(x,y,0) = x^2 - xy + y^2, & \text{in } D \end{cases}$$

Find $\max\limits_{D_T} u$ and $\min\limits_{D_T} u$.

Pb3: Solve the following problem

$$\begin{cases} u_t(x,y,t) - \Delta u(x,y,t) = 0, & \text{in } (0,\pi)\times(0,\pi)\times(0,+\infty) \\ u_x(0,y,t) = u_x(\pi,y,t) = 0, & \text{in } (0,\pi)\times[0,+\infty) \\ u(x,0,t) = u(x,\pi,t) = 0, & \text{in } (0,\pi)\times[0,+\infty) \\ u(x,y,0) = \sin y\cos 2x - \sin 2y\cos x \end{cases}$$

Problem # 4 For (P_2) above, with Neumann boundary conditions, we define two quantities:

- The quantity of heat $\varphi(t) = \int_D u(x,y,t)\,dx\,dy$
- The thermal energy $E(t) = \int_\Omega u^2(x,y,t)\,dx\,dy$

Show that $\varphi(t) \le \varphi(0)\,e^{-t}$ and $E(t)\le E(0)\,e^{-t}$.