Math 568 – 172

HW # 3

Problem #1. 1°) Suppose that $u \in C^2(\mathbb{R}^n)$ is harmonic such that $\lim_{|x| \to +\infty} |u(x)| = 0$. Use the maximum principle to show that $u \equiv 0$ (i.e $u(x) = 0, \forall x \in \mathbb{R}^n$).

2°) Discuss the uniqueness of the problem

$$\begin{cases} \Delta u = f, & \text{in } \mathbb{R}^n \\ \lim_{|x| \to +\infty} |u(x)| = 0 \end{cases}$$

Hint For 1°), you may consider the maximum principle in a large ball $B_R(0)$, $R > 0$ very large.

Problem #2 let $u(x,y)$ be a nonconstant function, which is harmonic in the disk $D = \{(x,y) / x^2+y^2 < R^2\}$, $R>0$. we define $f : [0,R) \longrightarrow \mathbb{R}$ by

$$f(r) = \max_{x^2+y^2 \le r^2} u(x,y).$$

Show that f is monotone increasing in $I = (0,R)$.

Problem #3. Consider the problem

$$\begin{cases} \Delta u(x,y) = 0 & \text{in } D \\ u(x,y) = y + 2yx & \text{on } \partial D \end{cases} \qquad y + 2xy$$

where $D = \{(x,y) \in \mathbb{R}^2 / x^2+y^2 < 1\}$

1°) write down the Laplace in polar coordinates (HW#2)

2°) Find u and write it in Cartesian coordinates.

Problem #4 Solve the problem

$$\begin{cases} \Delta u = 0, & 0 < x, y < \pi \\ u(x,0) = u(x,\pi) = 1, & 0 \le x \le \pi \\ u(0,y) = u(\pi,y) = 0; & 0 \le y \le \pi \end{cases}$$

---

problem #5 (Exercise 4.5 p 144 from our book)

Problem #6. Suppose that $u \in C^2(\Omega)$ satisfies

$$\begin{cases} -\Delta u + u^2 = 0 & \text{in } B \\ u(x,y,z) = xyz & \text{on } \partial B \end{cases}$$

where $B = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 < 1\}$

---

problem #7 Show that the problem

$$\begin{cases} -\Delta u + u = f & \text{in } \Omega \\ u|_{\partial\Omega} = 0 \end{cases}$$

has at most one solution. Ω is a bounded domain of $\mathbb{R}^n$.