Math 568_172
HW # 2

Problem #1 Solve the following problem

$$u_{xx} + 3u_{xy} - 4u_{yy} = xy, \quad -\infty < x, y < +\infty$$

$$u(x,x) = \sin x, \quad u_x(x,x) = 0$$

Problem # 2 Check if the operator given by

$$Lu = \sum_{1 \le i < j \le n} \frac{\partial^2 u}{\partial x_i \partial x_j}$$ is elliptic

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Problem 3 : in $\mathbb{R}^2$, the Laplace operator is given by $\Delta u = u_{xx} + u_{yy}$. Show that in polar coordinates,

$$\Delta u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}, \quad r > 0$$

Remember that $x = r\cos\theta$, $y = r\sin\theta$.

Problem #4 Given the equation

$$(1+x^2)^2 u_{xx} + u_{yy} + 2x(1+x^2) u_x = 0$$

1o) Show that the pde is elliptic

2o) Find the Canonical form

3o) Find the solution, if $u(0,y) = y$, $u_x(0,y) = 2y$