# Math 568_172

## HW # 1

**Problem #1** Solve the problem

$$u_x + (\sin x)\, u_y = y \,, \quad u(0,y) = 0$$

and Sketch the projected characteristic curves.

**Problem #2** (a) Solve the problem

$$y u_x + x u_y = 0 \,, \quad u(0,y) = e^{-y^2}$$

(a) In which region is $u$ uniquely defined.

**Problem #3** Solve the nonlinear problem

$$\begin{cases} (1+u)\, u_x + u u_y = 0 \,, & (x,y) \in \mathbb{R}^2 \\ u(0,y) = y \end{cases}$$

what happens when $x = 0$, $y = -1$ ?

**Problem #4** Find a solution of

$$\begin{cases} u u_x + u_y = -\frac{1}{2} u \\ u(x, 2x) = x^2 \end{cases}$$

Is it unique ?

**Problem #5** solve

$$\begin{cases} u_x + u_y - 2(x+y) z\, u_z = 0 \,, & (x,y,z) \in \mathbb{R}^3 \\ u(x,y,1) = e^{x^2+y^2} \end{cases}$$

and indicate the domain de definition of the Solution.