FINAL EXAM MATH 499
Special Topics Course in General Relativity
Semester 172

Time Allowed 3 Hours **Maximum Points: 140**

Q1. Given a Lorentzian metric $g_{ab} = (1,-1,-r^2,-r^2\sin^2\vartheta)$ answer the following:

a) What is its signature. **(3 points)**

b) Find the Ricci tensor components for this metric and justify your answer. **(2 Point)**

Q2. Assume that a tensor in 2-dimensional Cartesian space has components T^{11} and T^{12}. Find $T^{\hat{1}\hat{1}}$ and $T^{\hat{1}\hat{2}}$ under a coordinate transformation given by $x = r\cos\vartheta$ and $y = r\sin\vartheta$. **(5 Points)**

Q3. Consider a 2-dimensional metric $ds^2 = dt^2 - dx^2$. Find the form of this metric under the transformation defined by $t = X\sinh T$ and $x = X\cosh T$. **(5 Points)**

Q4. Solve geodesic equation in a 2-dimensional Cartesian metric $ds^2 = dt^2 - dx^2$. **(5 Points)**

Q5. Consider a metric given by $ds^2 = X^2 dt^2 - 4dX^2$. Solve this equation for the trajectory photons in this metric. **(10 Points)**

Q6. Give number of Killing vectors admitted by Minkowski metric? Identify these Killing vectors with conservation laws? **(10 Points)**

Q7. Show that the Killing vector $x\partial_y - y\partial_x$ represents a rotational symmetry. **(10 Points)**

Q8. For the metric $ds^2 = \phi(r)dt^2 - dx^2 - dy^2 - dz^2$, find $R^0{}_{101}$. **(5 Points)**

Q9. Use the second Bianchi identity $R^a{}_{b[cd;e]} = 0$ to construct Einstein tensor. **(10 Points)**

Q10. Write solution of the vacuum Einstein field equations in polar coordinates. Discuss its essential and coordinate singularities. What defines event horizon of this metric? **(15 Points)**

Q11. Briefly explain what are the three classical tests of general Relativity? Derive an expression that shows that the light grazing the surface of a gravitational source will be deflected. **(15 Points)**

Q12. Derive an expression for the Schwarzschild metric in Kruskal-Szekeres coordinates. **(20 Points)**

Q13. Assume a stress-energy-momentum tensor given by $T_{ab} = (\rho_o + p)u_a u_b - pg_{ab}$, with $u^a = e^{\nu/2}\delta^a_o$. Assuming ρ_o is a constant, $p(r)$ and $ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\vartheta^2 - r^2\sin^2\vartheta\, d\phi^2$, find equation of hydrostatic equilibrium arising as a consequence of $T^{1b}{}_{;b} = 0$. **(15 Points)**

Q14. Discuss singularities of a metric that represent an exact solution of the Einstein-Maxwell equations given by Reissner-Nordstrom metric. **(10 Points)**

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1. No calculators and mobile phones allowed.
2. Show your work to answer questions.

Question No	Points	Max Points
1		5
2		5
3		5
4		5
5		10
6		10
7		10
8		5
9		10
10		15
11		15
12		20
13		15
14		10
Total		140