King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 430
Final Exam – 2017–2018 (172)
Allowed Time: 180 minutes

Name: ______________________________

ID #: ____________________

Section #: ________ Serial Number: ___________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**

Problems:

Question #	Grade	Maximum Points
1		**12**
2		**10**
3		**6**
4		**8**
5		**10**
6		**12**
7		**12**
8		**10**
9		**10**
Total:		**90**

Q:1 (4 + 4 + 4 points) (a) Determine the set of points that satisfy the equation $|z+3| = |z-2|$.
(b) Find all the cube roots of $\sqrt{2} + i\sqrt{2}$.
(c) Find all numbers z such that $e^{iz} = 100$.

Q:2 (6 + 4 points) (a) Prove that the function $f(z) = e^x \cos(y) + ie^x \sin(y)$ is entire, and find its derivative in terms of z.
(b) Show that $u(x, y) = 3x^2y - y^3 - x$ is harmonic in the whole plane.

Q:3 (6 points) Evaluate $\int_C \dfrac{1+z}{z}\, dz$, where C is the right half of the circle $|z| = 1$ from $z = -i$ to $z = i$.

Q:4 (8 points) Use Cauchy's integral formula for derivatives to evaluate

$$\int_{|z|=1} \frac{z-1}{z^4 + 3iz^3} \, dz.$$

Q:5 (6 + 4 points) (a) Expand $f(z) = \dfrac{1}{z(z-1)}$ in a Laurent series valid for

$$(i)\ \ 1 < |z|, \qquad (ii)\ \ 0 < |z-1| < 1.$$

(b) Show that $z = 0$ is an essential singularity of $f(z) = z^3\ e^{\frac{1}{z}}$.

Q:6 (6 + 6 points) (a) Use Cauchy's residue theorem to evaluate

$$\int_{|z|=5} \frac{\sin(\mathrm{z})}{\mathrm{z}^2 - 4} \mathrm{dz}.$$

(b) Show that $\int_0^{2\pi} e^{\cos\theta} \cos(n\theta - \sin\theta) d\theta = \dfrac{2\pi}{n!}$.

Q:7 (6 + 6 points) (a) Let $|F(z)| \leq \frac{M}{R^k}$ for $z = Re^{i\theta}$, where $k > 0$ and M are constants. Prove that

$$lim_{R\to\infty} \int_C e^{imz} F(z) dz = 0,$$

where C is the semicircular arc of radious R $(0 \leq \theta \leq \pi)$ and m is a constant.
(b) Define meromorphic function. Define Argument principle and use it to evaluate

$$\int \frac{f'(z)}{f(z)} dz,$$

where $$f(z) = \frac{(z-8)^2 z^3}{(z-5)^4 (z+2)^2 (z-1)^5}.$$

Q:8 (6 + 4 points) (a) Define conformal mapping. Find all points where the complex mapping $f(z) = \sin(z)$ is conformal.
(b) Find the images of the points 0, $1 + i$, i and ∞ under the Mobius transformation

$$w = \frac{(2z + 1)}{(z - i)}.$$

Q:9 (6 + 4 points) (a) Let $f_1(z) = \dfrac{z+2}{z+3}$, $f_2(z) = \dfrac{z}{z+1}$. Find $f_1^{-1}(f_2(z))$.

(b) Show that $w = \dfrac{i(1-z)}{1+z}$ maps the unit disk $|z| < 1$ one to one and onto the upper kalf plane $Im(w) > 0$.