**King Fahd University of Petroleum & Minerals**
**Department of Mathematics & Statistics**
**Math 430 Exam 03**
**The Second Semester of 2017-2018 (172)**
**Time Allowed: 90 Minutes**

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Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

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- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

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| Question # | Marks | Maximum Marks |
|---|---|---|
| 1 | | 9 |
| 2 | | 11 |
| 3 | | 8 |
| 4 | | 10 |
| 5 | | 13 |
| Total | | 51 |

**Q1:** (2 + 7 points) (a) Define simply connected domain.

(b) State the Cauchy's integral theorem and use it to evaluate

$\int_C \frac{1}{z^4-1}dz$, where $C$ is the circle $x^2+(y-1)^2=1$.

**Q2:** (4 + 7 points) (a) Use Cauchy's integral formula(s) to evaluate

$$\int_{|z|=2} \frac{z^3+3}{z(z-i)^2}dz,$$

(b) State and prove Cauchy's inequality.

**Q3:** ( 8 points) State the maximum modulus theorem and use it to find the maximum value of $|(z^2 + 3z - 1)|$ in the disk $|z| \leq 1$.

**Q4:** (5 + 5 points) (a) Fnd the radius convergence of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}(z-1-i)^k$. Also, find a domain in which convergence absolutely holds.

(b) Find an explicit formula for the analytic function $f(z)$ that the Maclaurin expansion $\sum_{k=1}^{\infty} k^2 z^k$ (Write all steps).

**Q5:** (5 + 8 points) (a) Let $f(z) = \sum_{k=1}^{\infty} \frac{k^3}{3^k} z^k$. Compute $\int_{|z|=1} \frac{f(z)}{z^7}dz$,

(b) Compute the first four terms in the Taylor series of $f(z) = \frac{\cos(z)}{z^{12}+1}$ about $z_0 = 0$. Also , compute the coefficient of the $z^{12}$ term in the Taylor series of $f(z)$ about 0.