King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Major Exam II
The Second Semester of 2017-2018 (172)
Time Allowed: 90 Minutes

Name: _____ ID#: _____

Section: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		11
2		17
3		8
4		9
5		6
Total		51

Q1: (6 + 5 points) (a) Show that the function $f(z) = e^{x^2-y^2}[\cos(2xy) + i\ \sin(2xy)]$ is entire and find its derivative.
(b) Find a harmonic conjugate of $u(x, y) = \ln|z|$ for Re $z > 0$.

Q2: (3 + 4 + 5 + 5 points) (a) Find all the poles and their multiplicities for $R(z) = \frac{(3z+3i)(z^2-4)}{(z-2)(z^2+1)^2}$.

(b) Write (i) $\cosh(\frac{i\pi}{2})$ and (ii) $\cos(1 - i)$ in the form $a + ib$.

(c) (i) Find all values of $(1 + i)^3$.

(ii) Find all values of z satisfying $\sin\ z = 2$.

(d) Derive the identity $\sec^{-1}$ z $= -$i $\ln[\frac{1}{z} + (\frac{1}{z^2} - 1)^{\frac{1}{2}}]$

Q3: (5 + 3 points) (a) Parametrize the contour consisting of the perimeter of the square with vertices $-1 - i, 1 - i, 1 + i$ and $-1 + i$ traversed once in that order.

(b) Find the length of the contour represented by $z(t) = 5\ e^{3it},\ 0 \le t \le \pi$.

Q4: (4 + 5 points) (a) Let $f(z)$ be continous on $[a, b]$ and $F'(t) = f(t)$ for all t in $[a, b]$. Then show that $\int_a^b\ f(t)\ dt = F(b) - F(a)$.

(b) Compute $\int_\tau\ (|z - 1 + i|^2 - z)dz$ along the semicircle $z = 1 - i + e^{it},\ 0 \le t \le \pi$.

Q5: (6 points) State the **ML-inequality** and use it to find an upper bound for evaluate $|\int_C\ \frac{e^z}{z^2+1}dz|$, where C is the circle $|z| = 2$ traversed once in the counterclockwise direction.