King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430   Major Exam I
The Second Semester of 2017-2018 (172)
                                        Time Allowed: 90 Minutes

Name: _____   ID#: _____

Section/Instructor: _____   Serial #: _____

- Mobiles and calculators are not allowed in this exam.

- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		8
2		8
3		13
4		8
5		14
Total		51

(Q1:) (4 + 4)

(a) Solve the following system for z_1 and z_2 and write your answer in standard form $a + ib$:

$$3z_1 + (1 - i)z_2 = 2 - 3i$$

$$(1 + 2i)z_1 + iz_2 = 1.$$

(b) Show both analytically and graphically that $|z - 1| = |\bar{z} - 1|$.

(Q2:) (4 + 4)

(a) If z_1 and z_2 are arbitrary complex numbers, then show that

$$|z_1 + z_2| \le |z_1| + |z_2|$$

by using the identities $(|z|^2 = z\bar{z}, Re\ z = \frac{z + \bar{z}}{2}, |Re\ z| \le |z|)$

(b) Express $\dfrac{1 + i}{\sqrt{3} - i}$ in terms of polar forms.

(Q3:) (5 + 4 + 4)

(a) Find all solutions to the equation $z^2 + (1 + i)z + 5i = 0$ in terms of $a + ib$.

(b) Solve the equation $z^8 = 1$.

(c) Evaluate $(-\sqrt{3} - i)^{30}$.

(Q4:) (4 + 4)

(a) Describe the set of ppoints z in the complex plane that satisfy $|z| = |z - i|$.

(b) Let S be a subset of C. Prove that S is closed if and only if its complement $C - S$ is an open set.

(Q5:) (5 + 5 + 4)

(a) Show that $w = \frac{1}{2}(z + \frac{1}{z})$ maps the circle $|z| = r$ $(r > 0, r \ne 0)$ onto the ellipse.

(b) Find the image of the right half plane $Re(z) \ge 1$ under the mapping $w = (-1+i)z - 2 + 3i$ and represent the mapping graphically.

(c) Use $\epsilon - \delta$ definition to show that $\lim_{z \to i} z^2 = -1$