MATH411, Section 1
Spring 2018, Term 172

Final Exam

ID:

Name:

Answer the following questions.
Time: 120 minutes.

1. (20 pts) Identify and determine the nature of the critical points of $f(x,y) = \cos x \sin y$.

2. (40 pts) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$, be defined as

$$f(x,y) = \left(x^2 - y^2, 2xy\right).$$

(a) Prove that f is one-to-one for all $(x,y) \in \mathbb{R}^2$ such that $y < 0$.

(b) Find $D_{f^{-1}}(\frac{3}{4}, 1)$.

3. (40 pts) Consider the following system of equations

$$\begin{array}{ccccccc} xz^2 & + & 2zu^2 & + & yv^2 & = & 4 \\ x^2z & + & z^3u & + & y^2v & = & 3 \end{array}$$

(i) Show that this system has a unique solution $(u,v) = f(x,y,z) = (f_1(x,y,z), f_2(x,y,z)$ near the point $(1,1,1,1,1)$.

(ii) Find $D_f(1,1,1)$.

4. (40 pts) Let $R \subset \mathbb{R}^n$ be a rectangle, $f : R \to \mathbb{R}$ be a bounded function and $\mathcal{P}$ a partition of R. Prove that

$$U_{\mathcal{Q}}(f) \leq U_{\mathcal{P}}(f),$$

where $\mathcal{Q}$ is a refinement of $\mathcal{P}$ and $U_{\mathcal{P}}(f), U_{\mathcal{Q}}(f)$ are the upper *Darboux* sums of f with respect to $\mathcal{P}$ and $\mathcal{Q}$ respectively.

5. (40 pts) Show that the function $\varphi : \mathbb{R}^2 \to \mathbb{R}^2$ defined as

$$\varphi(u,v) = \left(u, \frac{u+v}{2}\right)$$

is a change of variables in $\mathbb{R}^2$ and use it to evaluate

$$\int_{\Omega^* = \varphi(\Omega)} x^5(2y - x)e^{(2y-x)^2},$$

where

$$\Omega^* = \left\{(x,y) : \frac{x}{2} \leq y \leq \frac{x}{2} + 1, 0 \leq x \leq 2\right\}.$$

and Ω is a bounded simple set of $\mathbb{R}^2$.