MATH411, Section 1
Spring 2018, Term 172

Exam 2

ID:

Name:

Answer the following questions.

1. (20 pts) Let $f : \Omega \to \mathbb{R}^m$ where Ω is an open subset in $\mathbb{R}^n$, and $a \in \Omega$.

(a) Define, $D_f(a)$, the derivative of f at a.

(b) Show that $D_f(a)$ is unique, assuming that f is differentiable at a.

2. (20 pts) Let $f : \Omega \to \mathbb{R}$ where Ω is an open subset in $\mathbb{R}^n$, and $a, u \neq 0 \in \Omega$.

(a) Define, $D_u f(a)$, the directional derivative of f at a in the direction of u.

(b) Show that $|D_u f(a)|$ is maximum in the direction of the gradient of f at a.

3. (40 pts) For the functions $f_1, f_2 : \mathbb{R}^3 \to \mathbb{R}, f : \mathbb{R}^3 \to \mathbb{R}^2, g : \mathbb{R}^2 \to \mathbb{R}^3$ defined as

$$\begin{aligned} f_1(x,y,z) &= x - y, \\ f_2(x,y,z) &= xyz, \\ f(x,y,z) &= (f_1(x,y,z), f_2(x,y,z)), \\ g(x,y) &= (xy, x^2, y^2), \end{aligned}$$

compute

(i) $D_u f_1(1,1,1)$ where $u = (u_1, u_2, u_3) \in \mathbb{R}^3$,

(ii) $D_{(f \circ g)}(1,1)$ using the chain rule,

(iii) $H_{f_2}(1,1,1)$, the Hessian of f_2 at $(1,1,1)$,

(iv) $D^2_g(1,1)$, the second derivative of g at $(1,1)$.

4. (20 pts) About the point $(0,0)$, find the second order Taylor's formula for $f(x,y) = \cos x \cos y$ and show that

$$\lim_{(x,y)\to(0,0)} \frac{R_3(x,y)}{\|(x,y)\|^2} = 0$$

where $R_3(x,y)$ is second-order remainder. (Hint: compute $|R_3(x,y)|$)