King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 345: Group Theory

First Exam, Spring Semester 172 (120 minutes)

Jawad Abuhlail

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Show FULL details.

Q1. (10 points) Show that every subgroup of a cyclic group is cyclic.

Q2. (10 points) Give an example of a group G with an element g_n of order n for every $n \in \mathbb{N}$, and an element g of infinite order.

Q3. (10 points) Show that for each $n \in \mathbb{N}\backslash\{1\}$, the group $\mathbb{Z}_{2^{n-1}}$ has exactly n subgroups.

Q4. (10 points) Find the number of cyclic groups of order 4 in:
(a) D_6
(b) D_8

Q5. (10 points) Show that if H is a subgroup of S_n with $|H|$ odd, then $H \leq A_n$.

Q6. (10 points) Find the number of elements in A_5 of order:
(a) 2
(b) 5

Q7. (10 points) Show that S_4 is *not* isomorphic to D_{12}.

Q8. (30 points) Prove or disprove:

(a) The *centralizer* of any *non-identity* element in a group is Abelian.

(b) If G is a group with a *cyclic* subgroup H_n of order n for every $n \in \mathbb{N}$, then G is cyclic.

(c) Every infinite group has an infinite number of subgroups.

Bonus (5 points) Give an example of a group G and a *proper* subgroup H of G such that $G \simeq H$.

GOOD LUCK