Project2 Assignment for Math 321 (172)
Instructor: Muhammad Yousuf

Learning outcomes

In completing this project, the student will demonstrate the following skills and knowledge.

- Write and use Matlab scripts and functions for solving problems in engineering
- Identify the efficiency, accuracy, and limitations of the methods used
- Display, graph, analyze and interpret the output of the calculations

General instructions

- Do not work together on this project, everyone must submit his own unique report.
- Read the whole document carefully before you begin.
- You are welcome to contact me for any clarification.
- Get started early.

Matlab

- Write your own scripts for all used methods according to the algorithms in the textbook.
- Do not use Matlab built-in functions for the methods.
- Use Matlab with the double precision IEEE arithmetic.
- Use “disp” and “sprintf” to format the output.
- Use %g or %e format to format the errors and very small numbers.
- Use the various options for the “plot” and “subplot” functions to produce professional graphs.
- Use “hold” to combine graphs for comparison purpose. Use appropriate plot symbols and colors.

Report

- Use MS Word and its equation editor to typeset the report.
- Matlab figures can be saved as .jpg files and then pasted into the word file.
- The report should contain all your work including scripts, output, graphs, tables, observations, explanations, conclusions, etc.
- Structure the report in a clear, legible, attractive and concise fashion.
- The report should include:
    - Cover page with name, section number, and student id.
    - A page on how the project is related to your discipline.
- In all the tasks, include all appropriate mathematical part, graphs, and tables.

Submission

Submit printout of your report in the class on or before April 29, 2018.

Project2 Problem

Complete the following tasks for the problem written in front of your serial number.

Task 1

1. Use the Runge-Kutta method of order 4 to approximate the solution up to $t = 1$ using 5 steps.
2. On a single graph, plot the approximate solution and the exact solution. Compare.
3. Prepare a table showing all "t" values, the approximate solution, exact solution, and absolute error at all the time steps. Compare.

Task 2

1. Use the approximate solution **in Task 1** to construct the natural cubic interpolating splines.
2. On a single graph, plot all cubic splines with 15 points for each, exact solution with 90 points.

Task 3

1. Approximate the integral of the exact solution over the given interval using the composite Simpson's rule.
2. Approximate the integral of the exact solution over the given interval using the composite Simpson's rule and approximated solution values obtained **in Task 1**.
3. Approximate the integral of the exact solution over the given interval using the cubic splines obtained **in Task 2.**
4. Compare the three results and write your comments.

NOTE: All your final answers should be in 6 decimal places.

<u>Serial Numbers 1 - 5</u>

Consider the IVP

$$\frac{dy}{dt} = 2t(1 + y^2), \qquad y(0) = 0, \qquad 0 \le t \le 1.$$

The exact solution for this problem is

$$y(t) = \tan(t^2).$$

<u>Serial Numbers 6 - 10</u>

Consider the IVP

$$\frac{dy}{dt} = 1 + (t - y)^2, \qquad y(2) = 1, \qquad 2 \le t \le 3.$$

The exact solution for this problem is

$$y(t) = t + \frac{1}{1 - t}.$$

Serial Numbers 11 -15

Consider the IVP

$$\frac{dy}{dt} = t\sqrt{y} - y, \qquad y(2) = 2, \qquad 2 \leq t \leq 3.$$

The exact solution for this problem is

$$y(t) = \left(t - 2 + \sqrt{2}\, e\, e^{-\frac{t}{2}}\right)^2.$$

Serial Numbers 16 - 20

Consider the IVP

$$\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad y(1) = 1, \qquad 1 \leq t \leq 2.$$

The exact solution for this problem is

$$y(t) = \frac{t}{1 + \ln t}$$

Serial Numbers 21 -25

Consider the IVP

$$\frac{dy}{dt} = -ty + \frac{4t}{y}, \qquad y(0) = 1, \qquad 0 \leq t \leq 1.$$

The exact solution for this problem is

$$y(t) = \sqrt{4 - 3e^{-t^2}}.$$

Serial Numbers 26 - 30

Consider the IVP

$$\frac{dy}{dt} = \frac{2 - 2ty}{(1 + t^2)}, \qquad y(0) = 1, \qquad 0 \leq t \leq 1.$$

The exact solution for this problem is

$$y(t) = \frac{2t + 1}{1 + t^2}.$$