SOLUTION

MATH 321 HOMEWORK # 7

Exercise 3.3

Q:1 (a) $x_0 = 8.1$, $x_1 = 8.3$, $x_2 = 8.6$, $x_3 = 8.7$

$f(x_0) = 16.94410$, $f(x_1) = 17.56492$, $f(x_2) = 18.50515$,

$f(8.4) = ?$ $f(x_3) = 18.82091$

Degree One: $P_1(x) = a_0 + a_1(x - x_0)$

$$a_0 = f(x_0) = 16.94410$$

$$a_1 = f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{17.56492 - 16.9441}{8.3 - 8.1}$$

$$= 3.1041$$

$$P_1(x) = 16.94410 + 3.1041(x - 8.1)$$

$$= 3.1041x - 8.1991$$

$$P_1(8.4) = 17.8753$$

Degree Two: $P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$

$$a_0 = f(x_0) = 16.94410$$

$$a_1 = 3.1041$$

$$a_2 = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - a_1}{x_2 - x_0}$$

$$= 3.1341$$

$$P_2(x) = 0.06x^2 + 2.1201x - 4.1653$$

$$P_2(8.4) = 17.8771$$

Degree Three

$$P_3(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

$$= -0.00217x^3 + 0.11217x^2 + 1.6862x + 2.9608$$

$$P_3(8.4) = 17.8684$$

Q: 2 (a) $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$

$f(x_0) = 1$, $f(x_1) = 1.64872$, $f(x_2) = 2.71828$

$f(x_3) = 4.48169$

Approx. $f(0.43)$

$P_1(x) = 2.5949x + 1$, $P_1(0.43) = 2.1158$

$P_2(x) = 3.36677x^2 + 1.7532x + 1$

$P_2(0.43) = 2.3764$

$P_3(x) = 2.9121x^3 + 1.1827x^2 + 2.1172x + 1$

$P_3(0.43) = 2.3606$