SOLUTION

MATH 321-172 HOMEWORK #6

Exercise 3.1

Q:1a $x_0 = 0, \quad x_1 = 0.6 \quad x_2 = 0.9$

$f(x) = \cos x$

Degree 1 $L_{10} = \dfrac{x - x_1}{x_0 - x_1} = \dfrac{x - 0.6}{0 - 0.6} = -\dfrac{1}{0.6}(x - 0.6)$

$L_{11} = \dfrac{x - x_0}{x_1 - x_0} = \dfrac{x}{0.6}$

$P_1(x) = \dfrac{1}{0.6} x \, f(x_1) + \dfrac{-1}{0.6}(x - 0.6) f(x_0)$

$= -\dfrac{x}{0.6} + 1 + \dfrac{0.8253}{0.6} x$

$= -0.2911x + 1$

Absolute Error $= |\cos(0.45) - P_1(0.45)|$

$= 0.0314$

Degree 2

$L_{20}(x) = \dfrac{(x - 0.6)(x - 0.9)}{(0 - 0.6)(0 - 0.9)} = \dfrac{50}{27} x^2 - \dfrac{25}{9} x + 1$

$L_{21}(x) = \dfrac{(x - 0)(x - 0.9)}{(0 - 0.6)(0 - 0.9)} = -\dfrac{50}{9} x^2 + 5x$

$L_{22}(x) = \dfrac{(x - 0)(x - 0.6)}{(0.9 - 0)(0.9 - 0.6)} = \dfrac{100}{27} x^2 - \dfrac{20}{9} x$

$P_2(x) = 0.4311 x^2 - 0.03246x + 1$

$P_2(0.45) = 0.8981$

Absolute Error $= |\cos(0.45) - P_2(0.45)|$

$= 0.0023$

Q:1 c $\quad P_1(x) = 0.78334x$

Abs. Error $= |\ln(1.45) - P_1(0.45)|$

$= 0.019$

$P_2(x) = -0.2339x^2 + 0.9237$

Abs. Error $= |\ln(1.45) - P_2(0.45)|$

$= 0.0033$

HW # 6

Q:3 Find error bounds for

(1) $P_1(x) = -0.2911x + 1$ $\qquad f(x) = \cos x$

$$\text{Error} = \frac{f''(\xi(x))}{2!}(x-x_0)(x-x_1)$$

$$= -\frac{\cos(\xi(x))}{2}(x-0)(x-0.6)$$

Let $g(x) = x^2 - 0.6x$

$$g'(x) = 2x - 0.6 = 0 \Rightarrow x = 0.3$$

$$g(0.3) = -0.09$$

$$\text{max. Error} = \left|\frac{\cos(\xi(x))}{2}\right| |x^2 - 0.6x|$$

$$\leq \frac{1}{2}(0.09) = 0.045$$

(2) $P_2(x) = 0.4311x^2 - 0.03246x + 1$

$$\text{Error} = \frac{f'''(\xi(x))}{3!}(x-0)(x-0.6)(x-0.9)$$

Let $g(x) = x(x-0.6)(x-0.9) = x^3 - 1.5x^2 + 0.54x$

$$g'(x) = 3x^2 - 3x + 0.54 = 0$$

$x_1 = 0.19233$, ~~$x_2 = 2.8077$~~

$g(x_1) = 0.0555$, ~~$g(x_2) = 11.8245$~~

$$\text{Max. Error} \leq \frac{1}{6}(0.0555) = 0.00925$$

HW # 6

Q: 6(a) $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$

$f(0) = 1$ $f(0.25) = 1.64872$, $f(0.5) = 2.71828$

Approximate $f(0.43)$ $f(0.75) = 4.48169$

① $P_1(x)$ using x_1 and x_2

$P_1(x) = 2.5949x + 1$

$P_1(0.43) = 2.1158$,

$$\text{Error} = |e^{2(0.43)} - P_1(0.43)| = 0.2473$$

② $P_2(x)$ using x_0, x_1, x_2

$P_2(x) = 3.3667x^2 + 1.7532x + 1$

$P_2(0.43) = 2.3764$,

$$\text{Error} = |e^{2(0.43)} - P_2(0.43)| = 0.0132$$

③ $P_3(x)$ using all points

$P_3(x) = 2.9121x^3 + 1.1827x^2 + 2.1172x + 1$

$P_3(0.43) = 2.3606$,

$$\text{Error} = |e^{2(0.43)} - P_3(0.43)| = 0.0025$$

Q: 8a Look These Boxes

HW# 6

Q:9 $(0,0), (0.5, y), (1,3), (2,2)$

$$P(x) = \frac{8}{3}x^3y - \frac{16}{3}x^3 + 14x^2 - 8x^2y + \frac{16}{3}xy - \frac{17}{3}x$$

Coeff. of $x^3 = 6$

$$\frac{8y}{3} - \frac{16}{3} = 6$$

$$\frac{8}{3}y = 6 + \frac{16}{3} = \frac{34}{3}$$

$$y = \frac{34}{8} = \frac{17}{4}$$
