SOLUTION

MATH 321-172 HOMEWORK #4

Q:4 (2.2) $f(x) = x^4 + 3x^2 - 2$, $p_0 = 1$

(a) $x = \sqrt{\frac{2-x^4}{3}}$, $p_1 = 0.5774$, $p_2 = 0.7935$, $p_3 = 0.7311$, $p_4 = 0.7559$

(b) $x = \sqrt[4]{2-3x^2}$, $p_1 =$ No real p

(c) $x = \frac{2-x^4}{3x}$, $p_1 = 0.3333$, $p_2 = 1.9877$, $p_3 = -2.2822$, $p_4 = 3.6700$

(d) $x = \sqrt{\frac{2-3x^2}{x}}$, No Convergence

↙

$p_1 =$ No real p.

Q:8 (2.2) $x^3 - x - 1 = 0$, $Tol = 10^{-3}$, $p_0 = 1$, $0 \le x \le 1$.

$x = x^3 - 1$. No fixed point.

$x = (x+1)^{\frac{1}{3}}$ $p = 1.3243$, $itt = 4$.

Q:10 (2.2) $g(x) = 2^{-x}$, $\frac{1}{3} \le x \le 1$

$g(1) = 2^{-1} = \frac{1}{2} = 0.5$, $g(\frac{1}{3}) = 2^{-\frac{1}{3}} = \frac{1}{2^{\frac{1}{3}}} = 0.7937$

So $g(x) \in [\frac{1}{3}, 1]$

with $p_0 = 0.5$, $p = 0.6412$, $itt = 11$

Now $g'(x) = -2^{-x} \ln 2$

$|g'(\frac{1}{3})| = |-2^{-\frac{1}{3}} \cdot \ln 2| = 0.5502 < 1$

$|g'(1)| = |-2^{-1} \ln 2| = 0.3466 < 1$

$g''(x) = 2^{-x} \ln 2 \neq 0$ ✓

$K = 0.5502$

$$\frac{K^n}{1-K} \max\{0.5 - \tfrac{1}{3}, 1 - 0.5\} < 10^{-4}$$

$$\frac{(0.5502)^n}{1-0.5502}(0.5) < 10^{-4}$$

$$(0.5502)^n < \frac{10^{-4} \times 0.4498}{0.5} = 0.8997 \times 10^{-4}$$

$$n > \frac{\log(0.8997 \times 10^{-4})}{\log(0.5502)} = 15.5924$$

Estimated itt = 16, Actual itt = 11