SOLUTION

MATH 321 - 172 HOMEWORK # 2

Q:1 1.2 Compute the absolute error and relative error in approximation of p by p^*.

Use FOUR Decimal places.

(a) $p = \pi = 3.1416$

$p^* = \frac{22}{7} = 3.1428$

Absolute Error $= |p - p^*| = 0.0013 = 1.3 \times 10^{-3}$

Relative Error $= \dfrac{|p - p^*|}{|p|}$

$= 0.0004 = 4.0 \times 10^{-4}$

Q:4 1.2 Find the largest interval in which p^* must lie to approximate p with ralative error 10^{-4}.

$p = \pi$ $\qquad \dfrac{|p^* - p|}{|p|} < 10^{-4}$

$-|p| 10^{-4} < p^* - p < |p| 10^{-4}$

$\pi - \pi 10^{-4} < p^* < \pi + \pi 10^{-4}$

$3.14128 < p^* < 3.14191$

Q:5 (a) Compute $\frac{4}{5} + \frac{1}{3}$

1.2

(i) Exact $\frac{12+5}{15} = \frac{17}{15}$

(ii) Three digits chopping

$\frac{4}{5} = 0.80$, $\frac{1}{3} = 0.33$

$\frac{4}{5} + \frac{1}{3} = 1.13$

(iii) Three digits rounding

Same

(iv) Relative Error $= \dfrac{\left|\frac{17}{15} - 1.13\right|}{\frac{17}{15}}$

$= 3.0 \times 10^{-3}$

Q: 2 **1.3**

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

(a) $$e \approx \sum_{n=0}^{5} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120}$$

$$= 1 + 1 + 0.5 + 0.1666$$

$$+ 0.0416 + 0.0083$$

$$= 2.7166$$

(b) $$e \approx \sum_{j=0}^{5} \frac{1}{(5-j)!} = \frac{1}{5!} + \frac{1}{4!} + \frac{1}{3!} + \frac{1}{2!} + \frac{1}{1!} + \frac{1}{0!}$$

$$\frac{1}{5!} = \frac{1}{120} = 0.008333$$

$$\frac{1}{4!} + \frac{1}{5!} = \frac{1}{24} + 0.0083$$

$$= 0.041666 + 0.0083 = 0.0499$$

$$\frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} = \frac{1}{6} + \frac{1}{24} + \frac{1}{120}$$

$$= 0.1666 + 0.0499 = 0.2165$$

$$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} = 2.7165$$