1)Approximate the integral using the Trapezoidal rule

$$\int_0^{\pi/4} x \sin x \, dx$$

2)

.et $h = (b - a)/3$, $x_0 = a$, $x_1 = a + h$, and $x_2 = b$. Find the degree of precision of the quadrature ormula

$$\int_a^b f(x)\, dx = \frac{9}{4} h f(x_1) + \frac{3}{4} h f(x_2).$$

MATH-321 **Term-172** **ClassQuiz11**

Name:	
ID:	
Sec: 1(9:00-9:50) 2(10:00-10:50)	

Integration Formulas:

6. $\int_{x_0}^{x_1} f(x)\ dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] - \frac{h^3}{12} f^{(2)}(\xi)$

7. $\int_{x_0}^{x_2} f(x)\ dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5}{90} f^{(4)}(\xi)$

8. $\int_{a}^{b} f(x)\ dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{(b-a)h^4}{180} f^{(4)}(\xi)$

1)Use the **Midpoint method** to approximate the solution to the following initial-value problems $y' = \frac{y^2}{1+t}$, $1 \le t \le 2$, $y(1) = -2$ with $h = 0.5$

2)Use the **Runge-Kutta Order Four** to approximate $y(1.2)$
$y' = t + y$, $1 \le t \le 2$, $y(1) = 5$ with $h = 0.2$

MATH-321 **Term-172** **ClassQuiz15**

Name:
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Sec: 1(9:00-9:50) 2(10:00-10:50)

$$w_0 = \alpha,$$

$$k_1 = hf(t_i, w_i),$$

$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right),$$

$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right),$$

$$k_4 = hf(t_{i+1}, w_i + k_3),$$

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

Determine the **clamped** cubic spline $S(x)$ that interpolates the data $f(0)=0$, $f(1)=1, f(2)=0$, and $f'(0)=1$, $f'(2)=2$

MATH-321 Term-172 ClassQuiz14

Name:
ID:
Sec: 1(9:00-9:50) 2(10:00-10:50)

$$\begin{bmatrix} 2h_0 & h_0 & 0 & \cdots & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \ddots & & \vdots \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & \cdots & \cdots & 0 & h_{n-1} & 2h_{n-1} \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{h_0}(a_1-a_0)-3f'(a) \\ \frac{3}{h_1}(a_2-a_1)-\frac{3}{h_0}(a_1-a_0) \\ \vdots \\ \frac{3}{h_{n-1}}(a_n-a_{n-1})-\frac{3}{h_{n-2}}(a_{n-1}-a_{n-2}) \\ 3f'(b)-\frac{3}{h_{n-1}}(a_n-a_{n-1}) \end{bmatrix}$$

$$b_j = (a_{j+1}-a_j)/h_j - h_j(c_{j+1}+2c_j)/3$$
$$d_j = (c_{j+1}-c_j)/(3h_j)$$

1)Use Euler's method to approximate the solution of

$$y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1, \text{ with } h = 0.25$$

The actual solution to the initial-value problem is

$y(t) = \frac{1}{2}\sin 2t - \frac{1}{3}\cos 3t + \frac{4}{3}$ Compare the actual error at each step to the error bound.

MATH-321 Term-172 ClassQuiz13

<u>Name:</u>
<u>ID:</u>
<u>Sec</u>: 1(9:00-9:50) 2(10:00-10:50)

1) Use the Composite Simpson's rule with n=4 to approximate the following integral.

$$\int_0^{20} e^{2x} \sin(x^2)\, dx$$

2) Write a MATLAB code which approximate the integral using Composite Trapezoidal rule with n = 100.

$$\int_0^{20} e^{2x} \sin(x^2)\, dx$$

MATH-321 Term-172 ClassQuiz12

Name:
ID:
Sec: 1(9:00-9:50) 2(10:00-10:50)

Integration Formulas:

6. $\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] - \frac{h^3}{12} f^{(2)}(\xi)$

7. $\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5}{90} f^{(4)}(\xi)$

8. $\int_{a}^{b} f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{(b-a)h^4}{180} f^{(4)}(\xi)$