FINAL Exam 172

MATH 513

Quiz 1:
Expand

$$f(x) := \sin x,\ 0 < x < \pi$$

in Fourier cosine series.

Quiz 2:
For

$$f(x) := \begin{cases} \sin x,\ -\pi/2 < x < \pi/2 \\ 0,\ elsewhere \end{cases}$$

compute $F(w)$. Prove that $F(w) \to -1/4i$ as $w \to 1$. Prove that

$$\int_0^\infty \frac{s\cos(\pi s/2)}{1-s^2}\sin(sx)ds = \pi f(x)/2.$$

Quiz 3:
Find
a) $\mathcal{L}^{-1}\left(e^{-2s}/(s^2+1)\right)$
b) $\mathcal{L}^{-1}\left(\ln\frac{s+a}{s+b}\right)$

Quiz 4:
Consider

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial y^2},\ -\infty < x < \infty,\ t > 0 \\ u(x,0) = F(x),\ u_t(x,0) = G(x) = 0 \\ F(x) = \begin{cases} \sin x,\ -\pi < x < \pi \\ 0,\ elsewhere \end{cases} \end{cases}$$

Write $F(x)$ in terms of Heaviside functions and find the solution in terms of H.

Quiz 5: Let $I := \int_{(0,0,0)}^{(1,1,1)}(2xdx + 3y^2dy + 4z^3dz)$.
Prove that the system is conservative
Find the potential
Use the FTC to evaluate the integral

Quiz 6:
Solve

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 \le x \le a,\ 0 \le y \le b, \\ u(0,y) = 0,\ 0 \le y \le b \\ u(x,0) = 0,\ 0 \le x \le a, \\ \frac{\partial u}{\partial x}\big|_{x=a} = 0,\ 0 \le y \le b \\ u(x,b) = u_0 \sin\frac{\pi x}{2a},\ 0 \le x \le a. \end{cases}$$