Department of Mathematics and Statistics, KFUPM
Math 280, Term 172
Exam 1, February 25, 2018, Duration: 120 minutes
Name and ID

Exercise 1(10 points).
Use the **augmented matrix and reduced echelon form** to solve the following system.

$$\left(\begin{array}{ccccccc} x & - & y & + & z & = & 1 \\ 2x & + & y & - & z & = & -1 \\ 2x & + & 4y & + & 14z & = & 8 \end{array}\right)$$

Exercise 2(10 points).

(1) Under which conditions on a and b, the system (S) below has a unique solution.

$$\left(\begin{array}{ccccccc} x & - & y & + & z & = & 1 \\ 2x & + & y & - & az & = & -1 \\ 2x & + & 4y & + & 14bz & = & 1 \end{array}\right)$$

(2) **Without any calculations** show that the system (S_1) has no solution.

$$\left(\begin{array}{ccccccc} x & - & y & + & z & = & 1 \\ 2x & + & y & + & 15z & = & -1 \\ 2x & + & 4y & + & 28z & = & 1 \end{array}\right)$$

Exercise 3(15 points).
Recall that a square matrix A is symmetric if $A = A^T$ and skew symmetric if $A^T = -A$.
(1) Let A by any square matrix. Prove that $A = M - N$ where M is symmetric and N is skew-symmetric.

(2) Application: Write the matrix $\begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \\ 3 & 3 & 4 & 4 \\ 4 & 4 & 3 & 3 \end{pmatrix}$ as a difference of symmetric and skew-symmetric matrices.

Exercise 4 (10 points).

Use Gauss-Jordan Method to find the inverse (if it exists) of the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$

Exercise 5 (15 points).

Use Cramer method to solve the system (S): $\left(\begin{array}{ccccccc} x & + & 3y & + & z & = & 1 \\ 2x & + & y & + & z & = & 5 \\ -2x & + & 2y & - & z & = & -8 \end{array}\right)$

Exercise 6(15 points).

Let $N = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

(1) Find N^2, N^3 and N^n for every $n \geq 3$.
(2) Use question (1) to find A^n for every $n \geq 2$ [Hint: Write $A = N + I$].
(3) Use question (2) to find A^{101}.

Exercise 7(10 points).

Let $A = \begin{pmatrix} 2 & 3 & 2 \\ 2 & -1 & 4 \\ 3 & 2 & 0 \end{pmatrix}$.

(1) Find the adjoint matrix $adj(A)$ of A.

(2) Find the inverse of the matrix A using its adjoint.

Exercise 8(15 points).
(1) Let M and N be two non-singular matrices. Are $M+N$ and MN non-singular? Justify.
(2) Find all 3×3 matrices A such that $AB = BA$ for every 3×3 matrix B.