**KFUPM/ Department of Mathematics & Statistics/ 172/ MATH 260/ Quiz 2**

**Name:** **ID #:** **Serial #:**

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1. [10pts] Determine whether the functions $y_1 = x^4$ and $y_2 = x^4 \ln x$ are linearly independent solutions (on $(0,\infty)$) of the DE: $x^2y'' - 7xy' + 16y = 0$.

**Solution**

- The functions $y_1$ and $y_2$ are linearly independent because none of them is a linear combination (i.e. a scalar multiple in this case) of the other.

  [Another way is to check that the Wronskian $W = \begin{vmatrix} x^4 & x^4 \ln x \\ 4x^3 & 4x^3 \ln x + x^3 \end{vmatrix}$ of $y_1$ and $y_2$ is not equal to 0 at $x = 1$ (for example), hence they are linearly independent.]

- We have $y_1' = 4x^3$, $y_1'' = 12x^2$. Hence

$$x^2y_1'' - 7xy_1' + 16y_1 = x^2\left(12x^2\right) - 7x\left(4x^3\right) + 16x^4 = 0$$

  and so $y_1$ is a solution of the DE.

  Also, $y_2' = 4x^3 \ln x + x^3$, $y_2'' = 12x^2 \ln x + 4x^2 + 3x^2 = 12x^2 \ln x + 7x^2$. Hence

$$x^2y_2'' - 7xy_2' + 16y_2 = x^2\left(12x^2 \ln x + 7x^2\right) - 7x\left(4x^3 \ln x + x^3\right) + 16x^4 \ln x = 0$$

  and so $y_2$ is a solution of the DE.

2. [10pts] Solve the DE $y'' - 8y' + 17y = 0$.

**Solution**. This DE is homogeneous, linear and with real constant coefficients. Its characteristic equation is

$$r^2 - 8r + 17 = 0$$

i.e. $(r-4)^2 + 1 = 0$. The roots of the equation are $4 + i$ and $4 - i$.
The general solution is therefore

$$y = C_1e^{4x}\cos x + C_2e^{4x}\sin x.$$