King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Term 172 – Math 260

Homework 1

(The due date is on Thursday February 1, 2018 in class.)

(**Question** 1) If $y_1 = x\cos(\ln x)$ and $y_2 = x\sin(\ln x)$, then verify that $y_1 - y_2$ is a solution of the differential equation $x^2y'' - xy' + 2y = 0$.

(**Question** 2) Consider the differential equation $xy' - 2y = x^2$ with the initial condition $y(1) = 1$. Let $y = x^2(C + \ln x)$.
(a) Verify that y is a solution of the above differential equation.
(b) Find the value of the constant C so that y is a solution of the above initial value problem.

(**Question** 3) Solve the initial value problem $\dfrac{dy}{dx} = x\sqrt{x^2+7}$, $y(-3) = 30$.

(**Question** 4) Find the position function of a moving particle with the acceleration function $a(t) = \dfrac{1}{\sqrt{t+2}}$, the initial velocity $v_0 = 0$, and initial position $x_0 = 0$.

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Homework 2

(Sections 1.4 – 1.5 – 1.6)

(The due date is on Thursday February 8, 2018 in class.)

(**Question** 1) Solve the differential equation $\dfrac{dy}{dx} = 2x\cos^2 y$.

(**Question** 2) A certain city had a population of 25,000 in 1960 and a population of 30,000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. In which year, the population is going to reach 100,000?

(**Question** 3) Solve the initial value problem $xy' = 2y + x^3 \cos x$, $y(\pi/4) = 0$.

(**Question** 4) Solve the differential equation $\dfrac{dy}{dx} + y = 3\sin x$.

(**Question** 5) Solve the initial value problem $(x + \sin^{-1}(y))dx + \dfrac{x+y}{\sqrt{1-y^2}}dy$, $y(1) = 0$.

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Homework 3
(Sections 3.1 – 3.2)

(The due date is on Thursday February 15, 2018 in class.)

(**Question** 1) Solve the the linear system $\begin{cases} 2x+3y-z=1 \\ x+z=0 \\ -x+2y-2z=0 \end{cases}$.

(**Question** 2) Find the echelon form of the matrix $\begin{bmatrix} 1 & -1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 1 & -1 \\ 0 & 1 & 0 & 2 & 3 \end{bmatrix}$.

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Homework 4

(Sections 3.3 – 3.6)

(The due date is on <u>Thursday February 22, 2018 in class.</u>)

(**Question** 1) Using Gauss-Jordan method, solve the linear system $\begin{cases} x + 2y + 3z = 4 \\ 5x + 6y + 7z = 8 \\ 9x + 10y + 11z = 12 \end{cases}$.

(**Question** 2) If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then calculate A^{2018}.

(**Question** 3) If $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$, then find A^{-1}.

(**Question** 4) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$.

(a) Calculate $(A - B)(A + B)$ and $A^2 - B^2$. What do you observe?

(b) Find the matrix X so that $AX = B$.

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Homework 5
(Sections 4.1 – 4.5)

(The due date is on Thursday March 08, 2018 in class.)

(**Question** 1) Determine whether the following sets are vector spaces:

(a) $\{(x, y, z, t) \in \mathbb{R}^4 \mid x + z = 0, y + t = 0\}$.

(b) $\{(x, y, z) \in \mathbb{R}^3 \mid xyz = 0\}$.

(**Question** 2) Consider the vectors $u = (1, 2, 3)$, $v = (0, 1, 2)$, and $w = (-1, 0, 1)$ in $\mathbb{R}^3$.

(a) Write the vector $(1, 1, 1)$ as linear combination of the vectors u, v, and w.

(b) Determine whether the vectors u, v, and w form a basis of $\mathbb{R}^3$.

(**Question** 3) Find the solution space of the linear system $\begin{cases} x_1 + 5x_2 + 13x_3 + 14x_4 = 0 \\ 2x_1 + 5x_2 + 11x_3 + 12x_4 = 0 \\ 2x_1 + 7x_2 + 17x_3 + 19x_4 = 0 \end{cases}$

(**Question** 4) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix}$.

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Homework 6
(Sections 5.1 – 5.2)

(The due date is on Thursday March 15, 2018 in class.)

(**Question** 1) Let $f(x) = e^x$, $g(x) = \sin(\ln(x+1))$, $h(x) = \cos(\ln(x+1))$.
(a) Calculate the Wronskian of f, g, and h at $x = 0$
(b) Are f, g, and h linearly dependent or independent? Explain.

(**Question** 2) Consider the 3rd order linear differential equation $y^{(3)} - 5y'' + 8y' - 4y = 0$, and three functions .$y_1 = e^{2x}$, $y_2 = xe^{2x}$, and $y_3 = e^x$.
(a) Verify that y_1, y_2, and y_3 are solutions of the differential equation.
(b) Verify that y_1, y_2, and y_3 are linearly independent,
(c) Find the solution of the differential equation satisfying the initial conditions $y(0) = 0$, $y'(0) = 4$, and $y''(0) = 1$.

(**Question** 3) If $y_c = c_1 e^x \cos x + c_2 e^x \sin x$ is the complementary solution and $y_p = x + 1$ is the particular solution of the differential equation $y'' - 2y' + 2y = 2x$, then find the solution of this differential equation satisfying the initial conditions $y(0) = 4$ and $y'(0) = 8$.

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Homework 7
(Sections 5.3 – 5.5)

(The due date is on Thursday March 22, 2018 in class.)

(**Question** 1) Solve the initial value problem $y'' - 8y' + 25y = 0$, $y(0) = 3$, $y'(0) = -3$.

(**Question** 2) Find the general solution of the linear differential equation

$$y^{(4)} + 4y''' + 6y'' + 4y' = 0.$$

(**Question** 3) Find the general solution of $2y'' + 4y' + 5y = x^2$.

(**Question** 4) Find a particular solution of $y''' + y'' - 4y' - 4y = e^x\left((5-5x)\cos x + (2+5x)\sin x\right)$.

(**Question** 5) Find, **without calculating the coefficients**, the general form of the particular solution of $(D-1)^3(D^2-4) = x(e^x + e^{2x}) + e^{-2x}$.

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Homework 8
(Sections 6.1 – 6.3)

(The due date is on <u>Thursday April 12, 2018 in class.</u>)

(**Question** 1) Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 5 & 3 & -3 \end{bmatrix}$.

(a) Find the eigenvalues and the corresponding eigenvectors of the matrix A.

(b) Find a basis and the dimension of eigenspace of each eigenvalue.

(c) Explain why the matrix A is diagonalizable.

(d) Find a diagonal matrix D and an invertible matrix P so that $A = PDP^{-1}$.

(**Question** 2) Consider the matrix $B = \begin{bmatrix} -1 & -1 & 1 \\ -2 & 0 & -2 \\ 3 & -3 & 1 \end{bmatrix}$.

Use Cayley Hamilton Theorem, to calculate B^3 and B^{-1}.

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Homework 9
(Sections 7.1 – 7.2)

(The due date is on Thursday April 19, 2018 in class.)

(**Question** 1) Rewrite the differential equation $y^{(4)} + 4y''' + 6y'' + 4y' + y = 0$ as a system of first order differential equations.

(**Question** 2) Consider the system of 1st order differential equations $\begin{cases} x' = y \\ y' = x \end{cases}$.

Express the system as a single high order differential equation and solve it.

(**Question** 3) Write the system given below in the form of $X'(t) = A(t)X(t) + F(t)$.

$$\begin{cases} x_1'(t) = -5x_1(t) + 5x_2(t) + 4x_3(t) + e^t \\ x_2'(t) = -8x_1(t) \;+\; 7x_2(t) \;+\; \sin(t^2) \\ x_3'(t) = x_1(t) \qquad - \qquad 3x_3(t) \end{cases}$$

(**Question** 4) Consider the initial value problem $X'(t) = AX(t)$, $X(0) = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ where $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$. Let $X_1 = \begin{bmatrix} e^{6t} \\ e^{6t} \end{bmatrix}$, $X_2 = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$ be solutions of this system.

(a) Verify that X_1 and X_2 are linearly independent.
(b) Solve the initial value problem.

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Homework 10

(Sections 7.3 – 7.5)

(The due date is on <u>Thursday April 26, 2018 in class.</u>)

(**Question** 1) Solve the initial value problem $X'(t) = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix} X(t)$, $X(0) = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$.

(**Question** 2) Let $A = \begin{bmatrix} -15 & -7 & 4 \\ 34 & 16 & -11 \\ 17 & 7 & 5 \end{bmatrix}$ whose only eigenvector is $\lambda = 2$ and only eigenvalue is $v = \begin{bmatrix} 7 \\ -17 \\ 0 \end{bmatrix}$. Find matrices Q and J where J is the Jordan normal form of A and Q is an invertible matrix so that $A = QJQ^{-1}$.

(**Question** 3) Find the general solution of the system $X'(t) = \begin{bmatrix} 1 & 10 & -12 \\ 2 & 2 & 3 \\ 2 & -1 & 6 \end{bmatrix} X(t)$.

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Homework 11

(Sections 8.1 – 8.2))

(The due date is on <u>Thursday May 03, 2018 in class.)</u>
(This homework is optional.)

Question Consider the non homogenous system $X'(t) = AX(t)+F(t)$ where $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $F(t) = \begin{bmatrix} e^t \\ 0 \\ e^{-t} \end{bmatrix}$.

(1) Find the solution of the associated homogenous equation.
(2) Calculate e^{At}.
(3) Find a particular solution of the system.
(4) Find the general solution of the system.