King Fahd University of Petroleum and Minerals
Department of Mathematical & Statistics

MATH 260 Second Major Exam

March 29, 2018, At 6 PM Time Allowed: 100 min Term 172

Name: ______________________________

ID: ______________________ Sec: ____________ Serial number: ____________

Check that this exam has 10 questions.

Important Instructions:

1. No electronic device (such as calculator, mobile phone, smart watch) is
allowed in this exam.
2. Justify your answers no credit is given for (correct) answers not supported
by work.
4. Write clearly. Marks may be deducted for messy work.
5. Write your name, ID number and Section number on the exam paper.

Question	Marks	Out of
1		7
2		10
3		10
4		12
5		12
6		15
7		10
8		7
9		10
10		7

Q1. Find rank of the following matrix:

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ -1 & 4 & 5 & 3 \\ 2 & 6 & 5 & 8 \\ 3 & 9 & 6 & 12 \end{bmatrix}$$

Q2. Let $y(x) = c_1 e^{3x} + c_2 e^{-x}$ be general solution of a homogeneous, constant coefficient, second order linear differential equation. Find the corresponding differential equation.

Q3. Express the vector $(1, 0, 3)$ as a linear combination of the vectors $(1, 1, 1)$, $(1, 1, 2)$ and $(2, 1, 1)$.

Q4. Use the method of variation of parameters to find a particular solution of

$$y'' - 4y = xe^x.$$

Q5. Consider the 2nd order linear differential equation

$$y'' - 4y' + 9y = 4x + 5$$

If $y_p = x + 1$ is its particular solution and $y_c = e^{2x}(c_1 \cos 5x + c_2 \sin 5x)$ is its complementary function, then find the solution of this differential equation satisfying the conditions $y(0) = 2$ and $y'(0) = -8$.

Q6. Solve the IVP

$$y''' + 12y'' + 36y' = 0; \qquad y(0) = 0,\ y'(0) = -3,\ y''(0) = 0.$$

Q7. Find all possible values of A and B in $\mathbb{R}$ for which the vectors $v_1 = (0, 3, 0, 0)$, $v_2 = (1, 0, 2, 0)$, $v_3 = (2, 1, 0, 3)$, $v_4 = (3, A, B, 0)$ form a basis for $\mathbb{R}^4$.

Q8. Let V be the subspace of $\mathbb{R}^4$ consisting of all vectors (a, b, c, d) such that $a + 2c = d$. Find a basis for V and the dimension of V.

Q9. Determine, using Wronskian at $x = \pi/4$, if the functions $f(x) = x$, $g(x) = \ln(\sin x)$ and $h(x) = \ln(\cos x)$ are linearly independent on $0 < x < \pi/2$.

Q10. Write $f(x) = 5$ as a linear combination of the functions $g(x) = \sin^2 x$ and $h(x) = \cos 2x$.