KFUPM – Department of Mathematics & Statistics

MATH202 – Term 172

Quiz **3**A (Duration: 25 Minutes)

Name: solution ID #: Section: 16

Q1 (3.5 Points) Verify that the set of function $\{x, x^2, 1/x\}$ form a fundamental set of solutions of $x^3y''' + x^2y'' - 2xy' + 2y = 0$ on the interval $(0, \infty)$. Form the general solution.

$y_1 = x,\quad y_1' = 1,\quad y_1'' = y_1''' = 0.$

$x^3(0) + x^2(0) - 2x + 2x = 0$

$y_2 = x^2,\quad y_2' = 2x,\quad y_2'' = 2,\quad y_2''' = 0.$

$x^3(0) + 2x^2 - 4x^2 + 2x^2 = 0$

$y_3 = 1/x,\quad y_3' = -1/x^2,\quad y_3'' = 2/x^3,\quad y''' = -\frac{6}{x^4}$

$x^3\left(-\frac{6}{x^4}\right) + x^2\left(\frac{2}{x^3}\right) - 2x\left(-\frac{1}{x^2}\right) + \frac{2}{x}$

$= -\frac{6}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = 0$

So, $y_1 = x$, $y_2 = x^2$, $y_3 = 1/x$ are solutions of the given equation.

$$W(x, x^2, 1/x) = \begin{vmatrix} x & x^2 & 1/x \\ 1 & 2x & -1/x^2 \\ 0 & 2 & \frac{2}{x^3} \end{vmatrix} = \frac{4}{x} + \frac{2}{x} + \frac{2}{x} - \frac{2}{x} = \frac{6}{x} \neq 0$$

the set $\{x, x^2, 1/x\}$ is linearly independent.

Thus, $\{x, x^2, 1/x\}$ forms a fundamental set of solutions of the given equation.

General Solution:

$y = c_1 x + c_2 x^2 + c_3 \frac{1}{x}$

Q2 (3 Points) Verify that $y_{p_1} = 3e^{2x}$ and $y_{p_2} = x^2 + 3x$ are particular solutions of

$$y'' - 6y' + 5y = -9e^{2x} \quad \text{and} \quad y'' - 6y' + 5y = 5x^2 + 3x - 16,$$

respectively. Find the particular solutions of

$$y'' - 6y' + 5y = 10x^2 + 6x - 32 - e^{2x}$$

and

$$y'' - 6y' + 5y = 16 - 5x^2 - 3x + 3e^{2x}$$

a) Particular solution of

$y'' - 6y' + 5y = 10x^2 + 6x - 32 - e^{2x}$

$= 2(5x^2 + 3x - 16) + \frac{1}{9}(-9e^{2x})$

$\Rightarrow y_p = 2y_{p_2} + \frac{1}{9}y_{p_1}$

$= 2(x^2 + 3x) + \frac{1}{9}(3e^{2x})$

$= 2x^2 + 6x + \frac{e^{2x}}{9}$

b) particular solution of

$y'' - 6y' + 5y = 16 - 5x^2 - 3x + 3e^{2x}$

$= -(5x^2 + 3x - 16) - \frac{1}{3}(-9e^{2x})$

$\Rightarrow y_p = -y_{p_2} - \frac{1}{3}y_{p_1}$

$= -x^2 - 3x - e^{2x}$

Q3. (3.5 points) The function

$$y_1 = x^{1/2} \ln x$$

is a solution of $4x^2y'' + y = 0$. Find the general solution of the given differential equation on the interval $(0, \infty)$.

Solution: The standard form

$$y'' + \frac{1}{4x^2} y = 0.$$

So, $P(x) = 0$. We have

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$= x^{1/2} \ln x \int \frac{1}{x(\ln x)^2}\,dx$$

$$= x^{1/2} \ln x \left(\frac{-1}{\ln x}\right) = -x^{1/2}$$

The general solution is

$$y = C_1 x^{1/2} + C_2 x^{1/2} \ln x$$