KFUPM – Department of Mathematics & Statistics
MATH202 – Term 172
Quiz **2** A (Duration: 25 Minutes)

Name: Solution ID #: Section: 16

Q1 (6.5 Points) Solve the ODEs

a) $(e^x + y) + (2 + x + ye^y)y' = 0, \quad y(0) = 1$

Solution:

$(e^x+y)\,dx + (2+x+ye^y)\,dy = 0$

$M = e^x + y, \quad N = 2+x+ye^y$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$ (exact)

$f(x,y) = \int M\,dx$
$= \int (e^x+y)\,dx$
$= e^x + xy + g(y)$

$\frac{\partial f}{\partial y} = x + g'(y) = 2+x+ye^y$

$g'(y) = 2 + ye^y$

$g(y) = 2y + ye^y - e^y$

Thus, $f(x,y) = e^x + xy + 2y + ye^y - e^y$

The general solution is

$e^x + xy + 2y + ye^y - e^y = C$

b) $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$

Put $u = y - 2x + 3$

$\frac{du}{dx} = \frac{dy}{dx} - 2$

Thus,

$\frac{du}{dx} + 2 = 2 + \sqrt{u}$

$\frac{du}{dx} = \sqrt{u} \Rightarrow \frac{du}{\sqrt{u}} = dx$

$\Rightarrow 2\sqrt{u} = x + C$

$2\sqrt{y-2x+3} = x + C$

1.a) $y(0) = 1$

$1 + 2 + 1 - 1 = C$

$\Rightarrow C = 3$

The solution is

$e^x + xy + 2y + ye^y - e^y = 3$

Q2 (3.5 Points) A thermometer is taken from an inside room to the outside, where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes it reads 30° F. What is the initial temperature of the inside room?

$T_m = 5,$

$T(1) = 55, \quad T(5) = 30$

$\frac{dT}{dt} = k(T - T_m)$

$\Rightarrow T(t) = T_m + ce^{kt}$
$= 5 + ce^{kt}$

$T(1) = 55$

$\Rightarrow 5 + ce^k = 55 \quad \cdots (1)$

$T(5) = 30$

$\Rightarrow 5 + ce^{5k} = 30 \quad \cdots (2)$

from (1) we have

$c = \frac{55-5}{e^k} = \frac{50}{e^k}$

From (2)

$c = \frac{30-5}{e^{5k}} = \frac{25}{e^{5k}}$

So, $\frac{50}{e^k} = \frac{25}{e^{5k}} \Rightarrow e^{4k} = \frac{1}{2}$

$4k = \ln\frac{1}{2} = -\ln 2$

$\Rightarrow k = -\frac{1}{4}\ln 2$

$\Rightarrow c = 50e^{\frac{1}{4}\ln 2}$

Thus, $T_0 = 5 + c = 5 + 50e^{\frac{1}{4}\ln 2}$

$\approx 64.46\,^\circ F$

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Quiz **2** B (Duration: 25 Minutes)

Name: Solution ID #: Section: 16

Q1 (6.5 Points) Solve the ODEs

a) $(x+y)^2 + (2xy + x^2 - 3)\dfrac{dy}{dx} = 0, \quad y(1) = 1$

$(x+y)^2 dx + (2xy + x^2 - 3)\, dy = 0$

$M = (x+y)^2, \quad N = 2xy + x^2 - 3$

$\dfrac{\partial M}{\partial y} = 2x + 2y = \dfrac{\partial N}{\partial x}$

(exact)

$f(x,y) = \int M\, dx = \int (x^2 + 2xy + y^2)\, dx$

$= \dfrac{x^3}{3} + x^2 y + x y^2 + g(y)$

$\dfrac{\partial f}{\partial y} = x^2 + 2xy + g'(y)$

$= 2xy + x^2 - 3$

$\Rightarrow \quad g'(y) = -3 \Rightarrow g(y) = -3y$

The general solution is

$\dfrac{x^3}{3} + x^2 y + x y^2 - 3y = C$

$y(1) = 1 \Rightarrow \dfrac{1}{3} + 1 + 1 - 3 = C$

$\Rightarrow C = -\dfrac{2}{3}$

The solution is

$\dfrac{x^3}{3} + x^2 y + x y^2 - 3y = -\dfrac{2}{3}$

b) $x\, y' = y + \sqrt{x^2 - y^2}, \quad x > 0$

(Homogeneous)

put $y = ux$

$\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$

$x\left(u + x\dfrac{du}{dx}\right) = ux + \sqrt{x^2 - u^2 x^2}$

$\Rightarrow x^2 \dfrac{du}{dx} = x\sqrt{1-u^2}$

$\Rightarrow \dfrac{du}{\sqrt{1-u^2}} = \dfrac{dx}{x}$

$\Rightarrow \sin^{-1}(u) = \ln x + C$

$\Rightarrow u = \sin(\ln x + c)$

$\Rightarrow \dfrac{y}{x} = \sin(\ln x + c)$

$\Rightarrow y = x \sin(\ln x + c)$

Q2 (3.5 Points) A thermometer is taken from an inside room to the outside, where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes it reads 30° F. What is the initial temperature of the inside room?

See Quiz 2A (Page 1)