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(Q1) Solve the ODE $2\dfrac{dy}{dx} = (1-y^2)\tan x$.
Find a singular solution of the ODE, if it exists.

It is a separable ODE:

0.5 $$\frac{2\,dy}{1-y^2} = \tan x\,dx$$

0.5 $$\left[\frac{1}{1+y} + \frac{1}{1-y}\right] dy = \tan x\,dx$$

0.5 $$\ln(1+y) - \ln|1-y| = \ln \sec x + C_1$$

$$\ln\left|\frac{1+y}{1-y}\right| = \ln|\sec x| + C_1$$

0.5 $$\frac{1+y}{1-y} = e^{C_1}\sec x$$

or $$y = \frac{c\sec x - 1}{1 + c\sec x} \quad \cdots (*)$$

zeroes of $h(y) = 1-y^2$ are $y = \pm 1$. 0.5

$y=-1$ can be obtained from $(*)$ by putting $C=0$.

$y=1$ is a singular solution 0.5

(Q2) Determine a region R of the xy-plane for which the differential equation $(1+y^3)y' = x^2$ has a unique solution whose graph passes through a point (x_0, y_0) in R.

0.5 $$y' = \frac{x^2}{1+y^3}.$$
$$f(x,y) = \frac{x^2}{1+y^3},$$

0.5 $$\frac{\partial f}{\partial y} = -\frac{3x^2y^2}{(1+y^3)^2}$$

1 f and $\frac{\partial f}{\partial y}$ are not defined at $y=-1$.

So,

1 $$R = \{(x,y) \mid y < -1\} \text{ or } R = \{(x,y) \mid y > -1\}$$

(Q3) Solve the IVP $(1+x)y' + y = -4x\sin(x^2)$, $y(0) = 5$. Give the largest interval I on which the solution is defined.

0.5 $$y' + \frac{1}{1+x}y = \frac{-4x\sin x^2}{1+x}$$

$$P(x) = \frac{1}{1+x}$$

The integrating factor is

1 $$e^{\int P(x)\,dx} = 1+x.$$

We obtain

0.5 $$d[(1+x)y] = -4x\sin x^2$$

or $$(1+x)y = -4\int x\sin x^2\,dx = 2\cos x^2 + C$$

or 1 $$y = \frac{2\cos x^2 + C}{1+x}$$

0.5 $y(0) = 5 \implies C = 3$

The solution of the IVP is

$$y = \frac{2\cos x^2 + 3}{1+x},$$

0.5 $$-1 < x < \infty.$$

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(Q1) Solve the ODE $\frac{dy}{dx} = (y^2 - y)\tan x$.
Find a singular solution of the ODE, if it exists.

It is a Separable ODE:

$$\frac{dy}{y^2-y} = \tan x \, dx$$

$$\left[\frac{1}{y-1} - \frac{1}{y}\right] dy = \tan x \, dx$$

$$\ln\left|\frac{y-1}{y}\right| = \ln|\sec x| + C_1$$

$$\frac{y-1}{y} = c \sec x$$

or $y = \frac{1}{1 - c\sec x}$ .... (*)

Zeroes of $h(y) = y^2 - y$ are $y = 0$ and $y = 1$.

$y = 1$ can be obtain from (*) by choosing $c = 0$.

but, $y = 0$ can not be obtained from (*).

Thus, $y = 0$ is a Singular Solution

(Q2) Determine a region R of the xy-plane for which the differential equation $(x^2 + y^2)y' = y^2$ has a unique solution whose graph passes through a point (x_0, y_0) in R.

$$y' = \frac{y^2}{x^2+y^2}$$

$$f(x,y) = \frac{y^2}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{2yx^2}{(x^2+y^2)^2}$$

f and $\frac{\partial f}{\partial y}$ are not defined at $(0,0)$.

R is any region in $\mathbb{R}^2$ that does not contain the origin

Q2) Solve the IVP $(x-1)y' + y = 4xe^{x^2}$, $y(0) = 2$. Give the largest interval I on which the solution is defined.

$$y' + \frac{y}{x-1} = \frac{4xe^{x^2}}{x-1}.$$

$$P(x) = \frac{1}{x-1}$$

The integrating factor:

$$e^{\int P(x)\,dx} = x - 1$$

We have

$$d[(x-1)y] = 4xe^{x^2}$$

or

$$(x-1)y = \int 4xe^{x^2} dx = 2e^{x^2} + c$$

or

$$y = \frac{2e^{x^2} + c}{x-1}$$

$y(0) = 2 \Rightarrow c = -4$

So,

$$y = \frac{2e^{x^2} - 4}{x-1}$$

$$-\infty < x < 1$$