Math 202 – Quiz 3 (Term 172)

Name:

ID #:

Problem 1 (3 points):

Check that $y_1 = cosh3x$ and $y_2 = sinh3x$ form a fu lamental set of solutions to the differential equation $y'' - 9y = 0$.

$y_1' = 3\sinh 3x \;,\quad y_1'' = 9\cosh 3x$

So $\quad y_1'' - 9y_1 = 9\cosh 3x - 9\cosh 3x = 0$

$y_2' = 3\cosh 3x \;,\quad y_2'' = 9\sinh 3x$

So $\quad y_2'' - 9y_2 = 9\sinh 3x - 9\sinh 3x = 0$

So $\quad y_1, y_2$ are two solution

$$W = \begin{vmatrix} \cosh 3x & \sinh 3x \\ 3\sinh 3x & 3\cosh 3x \end{vmatrix} = 3\left(\cosh^2 3x - \sinh^2 3x\right) = 3 \neq 0$$

So $\quad y_1, y_2$ form a fundamental set of solutions

Problem 2 (2 points): Find, by inspection, a particular solution of the differential equation

$$y'' + 2y = -4x$$

A particular solution can be of the form $y_p = ax + b$

$\Rightarrow \quad y_p' = a$ and $y_p'' = 0$

Substitute in DE:

$$0 + 2(ax + b) = -4x$$

$$\Leftrightarrow \quad 2ax + 2b = -4x$$

$$\Rightarrow \quad a = -2, \quad b = 0$$

$$\therefore \quad \boxed{y_p = -2x}$$

Problem 3 (5 points):

a. Check that $y_1 = lnx, \ x > 0$, is a solution of the differential equation

$$xy'' + y' = 0$$

Sol. $\quad y_1' = \frac{1}{x}, \quad y_1'' = -\frac{1}{x^2}$

$$\Rightarrow \quad x\left(-\frac{1}{x^2}\right) + \frac{1}{x} = -\frac{1}{x} + \frac{1}{x} \equiv 0$$

is a solution

b. Use the reduction of order method to find a second linearly independent solution.

Answer

let's rewrite the equation as

$$y'' + \frac{1}{x} y' = 0$$

So, a second solution is $y_2 = u(x) y_1$ such that

$$u(x) = \int \frac{e^{-\int p(x)\,dx}}{y_1^2}\,dx, \quad p(x) = \frac{1}{x}$$

$$= \int \frac{e^{-\ln x}}{(\ln x)^2}\,dx = \int \frac{dx}{x(\ln x)^2}\,dx$$

If we take $v = \ln x$ then $dv = \frac{dx}{x}$

So, $\quad u(x) = \int v^{-2}\,dv = -v^{-1} = -\frac{1}{\ln x}.$

Therefore the second solution can be taken $\boxed{y_2 = -1}$

Rmk: $y_2 = c$, $c \neq 0$ can be a second solution

$$W(y_1, y_2) = \begin{vmatrix} c & \ln x \\ 0 & \frac{1}{x} \end{vmatrix} = \frac{c}{x} \neq 0.$$