Math 202 – Quiz 2 (Term 172)

Name: ID #:

Problem 1 (5 points):

Solve the following initial value problem

$$y' - (x + y + 1)^2 = 0, \qquad y(-1) = 0$$

Sol. let $u = x + y + 1$

then $u' = \frac{du}{dx} = 1 + y'$

$\Longrightarrow y' = u' - 1 = u^2 \Rightarrow u' = 1 + u^2$

$\Rightarrow \frac{du}{1+u^2} = dx$

$\Rightarrow \tan^{-1} u = x + c$

$\Rightarrow u = \tan(x+c) = x + y + 1$

$\Rightarrow y = \tan(x+c) - x - 1$

$y(-1) = \tan(c-1) \cancel{+1} \cancel{-1} = 0 \Rightarrow$

$\tan(c-1) = 0 \Rightarrow \boxed{c = 1}$

Thus $\boxed{y = \tan(x+1) - (x+1)}$

Problem 2 (5 points):

A radioactive substance of 100 milligrams has decreased by 3% after 6 hours. If the rate of decay is proportional to the present amount at time t, find the amount remaining after 36 hours.

<u>Sol.</u> $\frac{dA}{dt} = kA \,, \quad A(0) = A_0$

$$\Rightarrow A(t) = A_0 e^{kt}$$

$$t = 6 \Rightarrow A(t) = 97\% A = \frac{97 A_0}{100}$$

$$\Rightarrow \frac{97}{100} \cancel{A_0} = \cancel{A_0} e^{6k}$$

$$\Rightarrow k = \frac{\ln 0.97}{6}$$

Hence $A(t) = A_0 e^{t \frac{\ln 0.97}{6}}$

$$t = 36 \Rightarrow A(36) = 100 e^{6 \ln 0.97} = 100 e^{\ln (0.97)^6}$$

$$\boxed{A(36) = (100)(0.97)^6}$$