Math 202 – Quiz 1 (Term 172)

Name: **ID #:**

Problem 1 (4 points):

Solve the following initial value problem

$$2y' - ysinx = 2sinx, \qquad y\left(\frac{\pi}{2}\right) = 0$$

Sol. $y' - y\dfrac{\sin x}{2} = \sin x$

$\mu(x) = e^{-\int \frac{\sin x}{2} dx} = e^{\frac{\cos x}{2}}$

So the solution $y\mu = \int \sin x \, e^{\frac{\cos x}{2}} dx + C$

let $u = \dfrac{\cos x}{2}$, $du = -\dfrac{\sin x}{2} dx$

$\int \sin x \, e^{\frac{\cos x}{2}} dx = -2\int e^{u} du = -2e^{u} = -2e^{\frac{\cos x}{2}}$

Thus $y\mu = -2e^{\frac{\cos x}{2}} + C$

$\Rightarrow \boxed{y = -2 + Ce^{-\frac{\cos x}{2}}}$

$y\left(\frac{\pi}{2}\right) = -2 + C = 0 \Rightarrow C = 2$

$\therefore$ The solution of IVP is $y = -2 + 2e^{-\frac{\cos x}{2}}$

Problem 2 (3 points): Find a one-parameter solution of the equation

$$y' + \frac{2x}{tany} = 0$$

Sol. $\frac{dy}{dx} = \frac{-2x}{\tan y}$

$\Rightarrow \tan y \, dy = -2x \, dx$

$\frac{\sin y}{\cos y} dy = -2x \, dx$

$\Rightarrow -\ln|\cos y| = -x^2 + C$

$\Rightarrow \ln|\cos y| = x^2 + c$

$\Rightarrow |\cos y| = e^{x^2} \cdot e^{c}$

$\therefore \cos y = c e^{x^2}$, where c is a constant.

The solution is

$$\boxed{y = \cos^{-1}(c x^2)}$$

Problem 3 (3 points): Check that the following differential equation is not exact and find an integrating factor so that it becomes exact (**Do not solve it**)

$$y(x + y + 1)dx + (x + 2y)dy = 0$$

Sol. $M = y(x+y+1)$, $N = x+2y$

$$\frac{\partial M}{\partial y} = x + 2y + 1 , \quad \frac{\partial N}{\partial x} = 1$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \iff \text{DE is not exact.}$$

$$\frac{\mu'(x)}{\mu(x)} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{x+2y+1-1}{x+2y} = 1$$

So $\mu'(x) = \mu(x)$

$\Rightarrow \boxed{\mu(x) = e^{x}}$ is an integrating factor