MATH 202.14 (Term 172)

Quiz 6 (Sects. 8.2 & 8.3) Duration: 20min

Name: ID number:

1.)(4pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} X$.

2.)(3pts)Solve the homogeneous linear system $X' = \begin{pmatrix} -5 & 3 \\ -2 & -1 \end{pmatrix} X$.

3.)(3pts)Solve the system $X' = AX + \begin{pmatrix} 1 \\ t \end{pmatrix}$, given that $\Phi(t) = \begin{pmatrix} e^t & e^{-2t} \\ e^t & 2e^{-2t} \end{pmatrix}$ is a fundamental matrix of $X' = AX$.

1.) $\begin{vmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0$, $(\lambda-2)^2 = 0$, $\lambda = 2, 2$

$(A-2I)K = 0$, $\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $K\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

$X_2 = (Kt + P)e^{2t}$, $(A-2I)P = K$

$\left(\begin{array}{cc|c} -1 & -1 & 1 \\ 1 & 1 & -1 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$ $P\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\Rightarrow X_2 = \begin{pmatrix} t-1 \\ -t \end{pmatrix} e^{2t}$

$\boxed{X(t) = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} t-1 \\ -t \end{pmatrix} e^{2t}}$

$t \in (-\infty, \infty)$

2.) $\begin{vmatrix} -5-\lambda & 3 \\ -2 & -1-\lambda \end{vmatrix} = 0$, $\lambda^2 + 6\lambda + 11 = 0$

$\lambda = -3 \pm i\sqrt{2}$

$(A - (-3+i\sqrt{2})I)K = 0$

$\left(\begin{array}{cc|c} -2-i\sqrt{2} & 3 & 0 \\ -2 & 2-i\sqrt{2} & 0 \end{array}\right)$ $(-2-i\sqrt{2})x + 3y = 0$

$K\begin{pmatrix} -3 \\ 2+i\sqrt{2} \end{pmatrix}$

$K = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + i\begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$

$X_1 = \begin{pmatrix} -3\cos\sqrt{2}t \\ 2\cos\sqrt{2}t - \sqrt{2}\sin\sqrt{2}t \end{pmatrix} e^{-3t}$

$X_2 = \begin{pmatrix} -3\sin\sqrt{2}t \\ \sqrt{2}\cos\sqrt{2}t + 2\sin\sqrt{2}t \end{pmatrix} e^{-3t}$

$\underline{X(t) = c_1 X_1 + c_2 X_2}$, $t \in (-\infty, \infty)$

3.) $X = \underbrace{\Phi C}_{X_c} + \underbrace{\Phi \int \Phi^{-1} F}_{X_p}$

$\Phi^{-1} = \frac{1}{e^{-t}} \begin{pmatrix} 2e^{-2t} & -e^{-2t} \\ -e^t & e^t \end{pmatrix} = \begin{pmatrix} 2e^{-t} & -e^{-t} \\ -e^{2t} & e^{2t} \end{pmatrix}$

$\Phi^{-1}F = \begin{pmatrix} 2e^{-t} - te^{-t} \\ -2e^{2t} + te^{2t} \end{pmatrix}$

$\int \Phi^{-1}F = \begin{pmatrix} -2e^{-t} + (t+1)e^{-t} \\ -e^{2t} + \left(\frac{t}{2} - \frac{1}{4}\right)e^{2t} \end{pmatrix}$

$X_p = \Phi \int \Phi^{-1} F = \begin{pmatrix} \frac{3}{2}t - \frac{9}{4} \\ 2t - 7/2 \end{pmatrix}$