MATH 202.14 (Term 172)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: **ID number:**

1.)(5pts)Find 2 powers series solutions of the DE: $(x-3)y'' - xy' - y = 0$.
2.)(2pts)Find the indicial roots of the DE $x^2y'' - 2x(1-x^5)y' - (1-x^6)y = 0$ **at** $x = 0$.
3.)(3pts)Find a relation of recurrence satisfied by c_n**, where** $y = \sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}}$ **is solution of the DE** $2xy'' + 3y' + 2y = 0$.

1) $y = \sum_{n=0} c_n x^n$

$$(x-3)\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - x\sum_{n=1}^{\infty} c_n n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$-6c_2 - c_0 + \sum_{k=1}^{\infty}\Big[c_{k+1}(k+1)k - 3c_{k+2}(k+1)(k+2) - c_k(1+k)\Big]x^k = 0$$

$$\begin{cases} c_2 = -\dfrac{c_0}{6} \\ c_{k+2} = \dfrac{k\,c_{k+1} - c_k}{3(k+2)}, \quad k=1,2, \end{cases}$$

case 1: $\boxed{c_0 = 0,\ c_1 \neq 0}$

$c_2 = 0$

$c_3 = -\dfrac{c_1}{9}$, $c_4 = -\dfrac{c_1}{54}$

$\Rightarrow y = c_1\Big(\underbrace{x - \dfrac{x^3}{9} - \dfrac{x^4}{54} + \cdots}_{y_1}\Big)$

case 2 $\boxed{c_0 \neq 0,\ c_1 = 0}$

$c_3 = -\dfrac{c_0}{54}$

$y = c_0\Big(\underbrace{1 - \dfrac{x^2}{6} - \dfrac{x^3}{54} + \cdots}_{y_2}\Big)$

$\boxed{y = c_1y_1 + c_2y_2 \text{ is the general solution}}$

2.) $p(x) = -2(1-x^5)$; $q(x) = -(1-x^6)$

$r(r-1) - 2r - 1 = 0$, $r^2 - 3r - 1 = 0$

$r_1 = \dfrac{3-\sqrt{13}}{2}$, $r_2 = \dfrac{3+\sqrt{13}}{2}$

3.) $y = \sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}}$, $y' = \sum_{n=0}^{\infty} c_n(n-\tfrac{1}{2})x^{n-\frac{3}{2}}$

$y'' = \sum_{n=0}^{\infty} c_n(n-\tfrac{1}{2})(n-\tfrac{3}{2})x^{n-\frac{5}{2}}$

$$\Rightarrow \sum_{n=1}^{\infty} 2c_n(n-\tfrac{1}{2})(n-\tfrac{3}{2})x^{n-1} + \sum_{n=1} 3c_n(n-\tfrac{1}{2})x^{n-1} + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

$$\sum_{k=0}^{\infty} 2c_{k+1}(k+\tfrac{1}{2})(k-\tfrac{1}{2})x^k + \sum_{k=0} 3c_{k+1}(k+\tfrac{1}{2})x^k + \sum_{k=0}^{\infty} 2c_k x^k = 0$$

$$\boxed{c_{k+1} = \frac{-2c_k}{(k+1)(2k+1)},\quad k=0,1,2.}$$