MATH 202.14 (Term 172)
Quiz 4 (Sects. 4.5, 4.6 & 4.7) **Duration: 20min**

Name: **ID number:**

1.)(3pts)Annihilator of lowest degree of $f(x) = 1 - 2x^2 \sin^2 x$.
2.)(4pts)Solve the DE: $y'' - 3y' + 2y = \cos(e^{-x})$.
3.)(3pts)Solve the DE: $x^3y''' + 3x^2y'' + xy' - y = 0$.

1.) $\sin^2 x = \frac{1-\cos 2x}{2}$

$f(x) = \underbrace{1 - x^2}_{D^3 \cdot 1} + \underbrace{x^2 \cos 2x}_{(D^2+4)^3}$

$\Rightarrow D^3(D^2+4)^3(f(x)) = 0$

2.) $y_c = c_1 e^x + c_2 e^{2x}$

$W(e^x, e^{2x}) = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = e^{3x}$

$y_p = u_1 e^x + u_2 e^{2x}$,

$u_1' = -\frac{e^{2x}}{e^{3x}} \cos(e^{-x}) = -e^{-x}\cos(e^{-x})$

$\Rightarrow u_1 = -\int e^{-x}\cos(e^{-x})\,dx = \sin(e^{-x})$

$u_2' = \frac{e^x}{e^{3x}} \cos(e^{-x}) = e^{-2x}\cos(e^{-x})$

$\Rightarrow u_2 = \int e^{-2x}\cos(e^{-x})\,dx$

integration by parts

$u' = e^{-x}\cos(e^{-x}) \rightarrow u = -\sin(e^{-x})$

$v = e^{-x} \longrightarrow v' = -e^{-x}$

$u_2 = -e^{-x}\sin(e^{-x}) - \int e^{-x}\sin(e^{-x})\,dx$

$= -e^{-x}\sin(e^{-x}) - \cos(e^{-x})$

$\Rightarrow y_p = -e^{2x}\cos(e^{-x})$.

$y = c_1 e^x + c_2 e^{2x} - e^{2x}\cos(e^{-x})$

3.) $y = x^m$

$m(m-1)(m-2) + 3m(m-1) + m - 1 = 0$

$(m-1)[m^2 - 2m + 3m + 1] = 0$

$(m-1)(m^2 + m + 1) = 0$

$m = 1$

$m^2 + m + 1 = 0$

$\Rightarrow m = \frac{-1 \pm i\sqrt{3}}{2}$

$y = c_1 x + c_2 x^{-\frac{1}{2}}\cos\left(\frac{\sqrt{3}}{2}\ln x\right) + c_3 x^{-\frac{1}{2}}\sin\left(\frac{\sqrt{3}}{2}\ln x\right)$,